USE OF SEM PROGRAMS TO PRECISELY MEASURE SCALE RELIABILITY

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1. Introduction. Reliability analysis has been discussed extensively mainly by psychometricians since Cronbach (1951) proposed famous α, and it still receives much attention even now. Reliability analysis tends to be discussed within the structural equation modeling these days (e.g., Raykov 1997; Hancock & Mueller, 2000).

Consider a one-factor model with correlated unique factors:

$$X_i = \mu_i + \lambda_i f + u_i \qquad (i = 1, \ldots, p), \tag{1}$$

where $E(f) = E(u_i) = 0$, $V(f) = 1$, $\mathrm{Cov}(u_i, u_j) = \psi_{ij}$, $\mathrm{Cov}(f, u_i) = 0$ and $\mathrm{Cov}(u_i, u_j) = 0$ $(i \neq j)$. The scale score is defined as the total sum of X_i, i.e., $X = \sum_{i=1}^{p} X_i$. The scale (or composite) reliability is then defined as

$$\rho = \frac{V(\sum_{i=1}^{p} \lambda_i f)}{V(X)} = \frac{(\sum_{i=1}^{p} \lambda_i)^2}{(\sum_{i=1}^{p} \lambda_i)^2 + \sum_{i=1}^{p} \psi_{ii} + \sum_{i,j,i\neq j}^{p} \psi_{ij}}. \tag{2}$$

While the traditional reliability (test) theory assumes $\psi_{ij} = 0$ for $i \neq j$, the assumption may not hold for many empirical data sets, and the recent literature focuses on effect of the unique factor correlations upon the traditional reliability measure such as α and ρ without ψ_{ij}.

If there are many pairs of correlated unique factors, there may be (additional) common factors that can account for the correlations. Then, practitioners can think that the scale is not unidimensional and consider subscales. The problem is how to do it when they can not assume common factors behind the unique factors correlated. A typical case is where there is a common factor with only two indicators (Kano 1997). In such cases, they have to use the model in (1) to estimate the reliability through the formula (2).

Here we discuss pragmatic estimation of the reliability based on the expression ρ in (2). The serious drawback of the model (1) is nonidentifiability, so that one can not estimate parameters. The ψ_{ij} has $p(p+1)/2$ parameters, which is the same as the number of variances and covariances of the observed variables.

One way to estimate ψ_{ij} is to use the residual covariance matrix, that is, $\hat{\psi}_{ij} = s_{ij} - \hat{\lambda}_i \hat{\lambda}_j \quad (i \neq j)$, where s_{ij} is the sample covariance between X_i and X_j, and $\hat{\lambda}_i$ is the estimate in the usual one-factor analysis model, i.e., the model with $\psi_{ij} = 0$ $(i \neq j)$. The approach, however, does not work, because $\sum_{i,j,i\neq j}^{p} \hat{\psi}_{ij}$ is almost zero almost always. Estimation process tries to minimize the residuals and in many cases, this also minimizes the sum of residuals like in regression analysis.

Here we propose to perform the Lagrange Multiplier test for unique factor covariances sequentially, as suggested by Raykov (2001). For this, the SEM program EQS is useful (Bentler 1995). The LM option of the EQS as `/LMTEST` with `SET= PEE;` gives a list of the pairs of unique factors to be correlated. The statistically significant pair is released to be a free parameter, and the model is reestimated. In the talk we demonstrate the process of estimating ψ_{ij} and show updated reliability coefficients.

Key words: Cronbach' α, structural equation modeling, LM test.

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