P. D. E. Seminar on January 15, 2024

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Viscous Fluid Flow in Unbounded Domains with Moving Boundaries

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We consider the Navier-Stokes system modelling the flow of a viscous incompressible fluid in an unbounded domain with moving boundary $\partial\Omega(t)$ and Dirichlet boundary conditions. Via a coordinate transform we reduce the problem to a modified non-autonomous Navier-Stokes system

$$\partial_t u(t) + A(t)u(t) = P(t)F - P(t)u \cdot \nabla^{\phi(t)} u, \quad u(0) = u_0 \quad \text{in } \Omega_0$$

where $\Omega_0 \subset \mathbb{R}^n$ is a fixed reference domain. Here $A(t)$ is a t-dependent modified Stokes operator on Ω_0, $P(t)$ a modified Helmholtz projection, and $\nabla^{\phi(t)}$ denotes a $\phi(t)$-dependent gradient. These operators depend also on the spatial variable $\xi \in \Omega_0$

To solve the initial-boundary value problem or find time-periodic solutions in unbounded domains we have to construct the fundamental operator $\{U(t,s)\}$ of the non-autonomous system

$$\partial_t u(t) + A(t)u(t) = 0, \ t > s, \quad u(s) = u_0$$

in case that the operators $A(t)$ are not boundedly invertible. An important property are t-independent estimates of $A(t)$, e.g. Sobolev embeddings for fractional Stokes operators $A(t)^\theta$ with t-independent bounds. The adjoint operators $A(t)^*$ will be analyzed similarly. The final main aim is to get global-in-time estimates of the fundamental operator $\{U(t,s)\}$ and to establish sufficiently fast decay rates.

The focus of the talk is put on the half space $\mathbb{R}^n_+$ with compact perturbations. The results are based on joint papers with K. Tsuda (Kyushu Sangyo University, Fukuoka).