Workshop on Statistical modeling for stochastic processes and related fields (online)

This workshop aims to exchange information on the state-of-the-art in statistical modeling for stochastic processes and related fields from theoretical, methodological, and implementation points of view.

Dates:
September 27 (Mon) - 30 (Thu), 2021

Time:
From 17:00 to 19:30 (in Japan Standard Time (JST))

Online workshop:
The style of the workshop is online.

Organizer:
Kengo Kamatani (ISM, Tokyo)
Hiroki Masuda (Kyushu Uniuversity)
Nakahiro Yoshida (University of Tokyo)
Masayuki Uchida (Osaka University)

Sponsors:
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Program (in Japan Standard Time)

September 27 (Mon) Chair: Nakahiro Yoshida (University of Tokyo)

17:00 - 17:05 Welcome Address by Nakahiro Yoshida (University of Tokyo)

17:05 - 17:40 Nakahiro Yoshida (University of Tokyo)
Asymptotic expansion in volatility parametric estimation revisited

17:40 - 18:15 Yuliia Mishura (Taras Shevchenko National University of Kyiv)
High-Frequency Trading with Fractional Brownian Motion

18:20 - 18:55 Hiroki Masuda (Kyushu University)
On mixed-rates structure in Gaussian quasi-likelihood inference for Lévy SDE

18:55 - 19:30 Igor Cialenco (Illinois Institute of Technology)
A power variation approach to statistical analysis of discretely sampled semilinear SPDEs

September 28 (Tue) Chair: Hiroki Masuda (Kyushu University)

17:00 - 17:35 Maud Delattre (Universite Paris-Saclay, INRAE, MaIAGE)
Statistical inference for discretely observed stochastic differential equations with mixed effects

17:35 - 18:10 Lorenzo Mercuri (University of Milan)
yuima.PPR: New Developments for the Point Process in the YUIMA package

18:20 - 18:55 Alexei Kulik (Politechnika Wrocławska)
Approximation in law of Markov processes by non-linear regressions: analytic background and statistical applications

18:55 - 19:30 Ciprian A Tudor (Université Lille 1)
Drift parameter estimation for the stochastic wave equation with space-time white noise

September 29 (Wed) [Satellite] Chair: Kengo Kamatani (ISM, Tokyo)

14:55 - 15:30 Goda Takashi (University of Tokyo)
Multilevel Monte Carlo methods for efficient nested simulations

15:30 - 16:05 Xin Tong (National University of Singapore)
Can Algorithms Collaborate? The Replica Exchange Method and Its Spectral Gap

16:05 - 16:40 Alexandre H. Thiery (National University of Singapore)
Exploiting geometry for walking larger steps in Bayesian Inverse Problems

September 29 (Wed) Chair: Kengo Kamatani (ISM, Tokyo)

17:00 - 17:35 Kengo Kamatani (ISM, Tokyo)
Scaling limit analysis of some piecewise deterministic Markov processes

17:35 - 18:10 Björn Sprungk (TU Bergakademie Freiberg)
Robust sampling methods for Bayesian inverse problems with small observational noise

18:20 - 18:55 Tony Lelievre (Ecole des Ponts ParisTech)
Adaptive importance sampling methods

18:55 - 19:30 Alexandros Beskos (University College London)
Manifold Markov chain Monte Carlo methods for Bayesian inference in diffusion models

September 30 (Thu) Chair: Masayuki Uchida (Osaka University)

17:00 - 17:35 Masayuki Uchida (Osaka University)
Adaptive test for ergodic diffusion processes from discrete observations

17:35 - 18:10 Mathias Vetter (Christian-Albrechts-Universität zu Kiel)
Jump regressions revisited

18:15 - 18:50 Markus Bibinger (Julius-Maximilians-Universität Würzburg)
Inference on jumps in high-frequency order-price models with one-sided noise

18:50 - 19:25 Yasutaka Shimizu (Waseda University)
M-estimation based on quasi-processes from discrete samples of Lévy processes

19:25 - 19:30 Closing Address by Masayuki Uchida (Osaka University)

Asymptotic expansion in volatility parametric estimation revisited

Nakahiro Yoshida[1]

[1] University of Tokyo, E-mail: nakahiro@ms.u-tokyo.ac.jp

Abstract:

In estimation of a volatility parameter from sampled data in a finite time horizon, we revisit asymptotic expansion of the quasi-maximum likelihood estimator and the quasi-Bayesian estimator. This problem was approached by the author in 2010s with the martingale expansion. In this talk, the basic tools are the theory of asymptotic expansion for Skorokhod integrals, the quasi-likelihood analysis, and the order estimate of a polynomial of multiple Wiener integrals.

High-Frequency Trading with Fractional Brownian Motion

Paolo Guasoni[1]; Yuliya Mishura[2]; Miklós Rásonyi[3]

[1] Dublin City University, School of Mathematical Sciences, Glasnevin, Dublin 9, Ireland, and Boston University, Department of Mathematics and Statistics, 111 Cummington Street, Boston, MA 02215, USA

[2] Taras Shevchenko National University of Kyiv, e-mail: myus@niv.kiev.ua

[3] Alfréd Rényi Institute of Mathematics, Budapest, Hungarian Academy of Sciences

Abstract: In the high-frequency limit, conditional expected increments of fractional Brownian motion converge to a white noise, shedding their dependence on the path history and the forecasting horizon, and making dynamic optimization problems tractable. We find an explicit formula for locally mean-variance optimal strategies and their performance for an asset price that follows fractional Brownian motion. Without trading costs, risk-adjusted profits are linear in the trading horizon and rise asymmetrically as the Hurst exponent departs from Brownian motion, remaining finite as the exponent reaches zero while diverging as it approaches one. Trading costs penalize numerous portfolio updates from short-lived signals, leading to a finite trading frequency, which can be chosen so that the effect of trading costs is arbitrarily small, depending on the required speed of convergence to the high-frequency limit.

On mixed-rates structure in Gaussian quasi-likelihood inference for Lévy SDE

Hiroki Masuda
Kyushu University
E-mail: hiroki@math.kyushu-u.ac.jp

Abstract:

In this talk, we are concerned with statistical inference for an ergodic solution to a stochastic differential equation (SDE) driven by a general Lévy process. We suppose that the trend and scale coefficients of the SDE are parametrized by finite-dimensional unknown parameters, and that the SDE is observed at high frequency over a long time period. Our motto here is to develop explicit and theoretically guaranteed procedures for estimation and selection, without pursuing theoretical efficiency.

We will begin with a brief overview of the existing literature, and then present a recent study in an AIC-type information criterion. Several easy-to-compute statistics for both estimation (quasi-likelihood estimator) and selection (quasi-information criterion) will be given in somewhat non-standard forms with partially, yet significantly and quantitatively reflecting the non-Gaussian character of the driving Lévy process. The whole story is based on the Gaussian quasi-likelihood constructed through the small-time (fake) Gaussian approximation of the transition probability. In particular, in both estimation and selection, we clarify the asymptotic effects of the mixed-rates structure of the Gaussian quasi-likelihood random fields, revealing essential differences from the case of diffusion.

This talk is, for the most part, based on the collaborations with Dr. Shoichi Eguchi (Osaka Institute of Technology, Japan) and Dr. Yuma Uehara (Kansai University, Japan).

A power variation approach to statistical analysis of discretely sampled semilinear SPDEs

Igor Cialenco[1]; Hyun-Jung Kim [2]; Gregor Pasemann [3]

[1] Illinois Institute of Technology, E-mail: cialenco@iit.edu
[2] University of California Santa Barbara, E-mail: hjkim@ucsb.edu
[3] Gregor Pasemann, E-mail:pasemann@math.tu-berlin.de

Abstract: Motivated by problems from statistical analysis for discretely sampled SPDEs, we derive central limit theorems for higher order finite differences applied to stochastic process with arbitrary finitely regular paths. We prove a new central limit theorem for some power variations of the iterated integrals of a fractional Brownian motion (fBm) and consequently apply them to estimation of the drift and volatility coefficients of semilinear stochastic partial differential equations driven by an additive Gaussian noise white in time and possibly colored in space. In particular, we show that approximating naively derivatives by finite differences in certain estimators may introduce a nontrivial bias that we compute explicitly. We will illustrate the theoretical results by numerical experiments.

Statistical inference for discretely observed stochastic differential equations with mixed effects

Maud Delattre[1]; Valentine Genon-Catalot[2]; Catherine Larédo[3]

[1] Université Paris-Saclay, INRAE, Jouy-en-Josas, France , E-mail: maud.delattre@inrae.fr

[2] Université de Paris, MAP5, Paris, France

[3] Université Paris-Saclay, INRAE, Jouy-en-Josas, France

Abstract:

Mixed effects models are popular tools for analyzing longitudinal data from several individuals simultaneously. Individuals are described by the same structural model and the inclusion of some random effects enables the description of the inter-individual variability which is inherent to the data. In stochastic differential equations with mixed effects, the structural model is a set of stochastic differential equations, which are usually observed at discrete times. The estimation of the population parameters, *i.e.* parameters of the random effects distribution, is a delicate problem because there is no closed form for the likelihood function. In order to characterize the statistical inference in such models, we consider a parametric framework with distributions leading to explicit approximate likelihood functions. We investigate the properties of the resulting estimators under the asymptotic framework where both the number of sample paths and the number of observations per path tend to infinity and where observations are collected within a fixed time interval. The estimation methods are assessed on various simulated data sets and then applied to real neuronal data.

yuima.PPR: New Developments for the Point Process in the YUIMA package.

Lorenzo Mercuri[1]

[1] University of Milan, email: lorenzo.mercuri@unimi.it

Abstract:

In this talk we present and discuss yuima classes and methods that allow the user to simulate and estimate a Point Process Regression Model (PPR). The PPR model can be seen as a generalization of a self-exciting point process, since it is possible to consider external covariates that explain the behaviour of the intensity process. To manage a PPR model, two new objects have been introduced. The first object belongs to yuima.PPR-class, contains the mathematical structure of a PPR model and eventually the dataset. The last object belongs to the yuima.PPR.qmle-class and is filled with a dataset, the model description and the estimated parameters obtained from the considered dataset. We discuss also how to use these objects to manage the CARMA-Hawkes process recently proposed in actuarial science to model the insurance claims. The CARMA-Hawkes process is a generalization of the standard Hawkes where the exponential kernel is substituted by the CARMA kernel. The main advantage of this model is its ability to reproduce a more complex dependence structure compared with the autocovariance generated by the Hawkes process.

Approximation in law of Markov processes by non-linear regressions: analytic background and statistical applications

Alexei Kulik[1]

[1] Wrocław University of Science and Technology, kulik.alex.m@gmail.com

Abstract: We will discuss the analytic methods suitable for a representation of the heat kernel of diffusions and Lévy-type processes in the form of an explicit kernel which corresponds to a certain non-linear regression and a residual part subject to small time estimates. Such a representation is strongly motivated by various applications to statistics of stochastic processes discretely observed with high frequency of observations, which will be discussed as well.

Drift parameter estimation for the stochastic wave equation with space-time white noise

Ciprian Tudor[1]

[1] Université Lille 1, E-mail: Ciprian.Tudor@univ-paris1.fr

Abstract:
We study the quadratic variations (in time and in space) of the solution to the stochastic wave equation driven by the space-time white noise. We give their limit (almost surely and in $ L ^{2}(¥Omega)$) and we prove that these variations satisfy, after a proper renormalization, a Central Limit Theorem. We apply the quadratic variation to define and analyze estimators for the drift parameter of the wave equation.

Scaling limit analysis of some piecewise deterministic Markov processes

Kengo Kamatani[1]

[1] Institute of Statistical Mathematics, E-mail: kamatani@ism.ac.jp

Abstract:

Recently, piecewise deterministic Markov processes have gained interest in the Monte Carlo community in the context of scalable Monte Carlo integration methods. In this talk, we will discuss scaling limits for some piecewise deterministic Markov processes. We will describe these results using multiscale analysis, which is a useful technique for this purpose.

This is joint work with J. Bierkens (TU Delft) and G. O. Roberts (Warwick).

Robust sampling methods for Bayesian inverse problems with small observational noise

Björn Sprungk[1]

[1] TU Bergakademie Freiberg, E-mail :Bjoern.Sprungk@math.tu-freiberg.de

Abstract:

The Bayesian approach to inverse problems provides a rigorous framework for the incorporation and quantification of uncertainties in measurements, parameters and models. We are interested in designing numerical methods which are robust w.r.t. the size of the observational noise, i.e., methods which behave well in case of concentrated posterior measures. The concentration of the posterior is a highly desirable situation in practice, since it relates to informative or large data. However, it can pose a computational challenge for numerical methods based on the prior as reference measure. We propose to employ the Laplace approximation of the posterior as the base measure for numerical integration in this context. The Laplace approximation is a Gaussian measure centered at the maximum a-posteriori estimate and with a covariance matrix depending on the Hessian of the log posterior density. We discuss the convergence of the Laplace approximation to the posterior measure in the Hellinger distance and analyze the efficiency of Monte Carlo methods based on it. In particular, we show that Laplace-based importance sampling and Laplace-based Markov chain Monte Carlo methods are robust w.r.t. the concentration of the posterior for large classes of posterior distributions and integrands whereas prior-based sampling methods are not.

This is joint work together with Claudia Schillings (U Mannheim), Daniel Rudolf (U Göttingen), and Philipp Wacker (U Erlangen-Nürnberg).

Adaptive importance sampling methods

Tony Lelièvre[1]

[1] Ecole des Ponts ParisTech, E-mail: tony.lelievre@enpc.fr

Abstract:

We will present the general strategy of free energy adaptive importance sampling methods. These are very efficient algorithms to sample multimodal distributions with an information on the modes given by a so-called collective variable which indexes the modes. We will also discuss recent techniques to build this collective variable.

References:

- T. Lelièvre, M. Rousset and G. Stoltz, Long-time convergence of an Adaptive Biasing Force method, Nonlinearity, 21, 1155-1181 (2008).
- G. Fort, B. Jourdain, T. Lelièvre and G. Stoltz, Convergence and efficiency of adaptive importance sampling techniques with partial biasing, Journal of Statistical Physics, 171(2), 220-268 (2018).
- B. Jourdain, T. Lelièvre and P.A. Zitt, Convergence of metadynamics: discussion of the adiabatic hypothesis, to appear in Annals of Applied Probability.
- Z. Belkacemi, P. Gkeka, T. Lelièvre and G. Stoltz, Chasing Collective Variables using Autoencoders and biased trajectories, https://arxiv.org/abs/2104.11061 .

Manifold Markov chain Monte Carlo methods for Bayesian inference in diffusion models

Matthew M Graham[1]; Alexandre H Thiery[2]; Alexandros Beskos[3]

[1] Affiliation, E-mail: University College London, m.graham@ucl.ac.uk

[2] Affiliation, E-mail: National University of Singapore, a.h.thiery@nus.edu.sg

[3] Affiliation, E-mail: University College London, a.beskos@ucl.ac.uk

Abstract:

Bayesian inference for nonlinear diffusions, observed at discrete times, is a challenging task that has prompted the development of a number of algorithms, mainly within the computational statistics community. We propose a new direction, and accompanying methodology – borrowing ideas from statistical physics and computational chemistry – for inferring the posterior distribution of latent diffusion paths and model parameters, given observations of the process. Joint configurations of the underlying process noise and of parameters, mapping onto diffusion paths consistent with observations, form an implicitly defined manifold. Then, by making use of a constrained Hamiltonian Monte Carlo algorithm on the embedded manifold, we are able to perform computationally efficient inference for a class of discretely observed diffusion models. Critically, in contrast with other approaches proposed in the literature, our methodology is highly automated, requiring minimal user intervention and applying alike in a range of settings, including: elliptic or hypo-elliptic systems; observations with or without noise; linear or non-linear observation operators. Exploiting Markovianity, we propose a variant of the method with complexity that scales linearly in the resolution of path discretisation and the number of observation times. Example Python code is given at git.io/m-mcmc.

Adaptive test for ergodic diffusion processes from discrete observations

Masayuki Uchida[1]; Tetsuya Kwai[2]

[1] Graduate School of Engineering Science, Osaka University, MMDS, CREST

E-mail : uchida@sigmath.es.osaka-u.ac.jp

[2] Graduate School of Engineering Science, Osaka University

E-mail : kawai@sigmath.es.osaka-u.ac.jp

Abstract :

We consider statistical hypothesis testing problems for multidimensional ergodic diffusion processes based on high frequency data. Kitagawa and Uchida (2014, JSPI) investigated the joint testing problem for the null hypothesis with respect to (w.r.t.) both the diffusion and drift parameters. In this talk, we propose the two-step testing method for the diffusion and drift parameters as follows: (i) test the null hypothesis H_1 w.r.t. only the diffusion parameter versus the alternative hypothesis A_1 (not H_1). (ii) test the null hypothesis H_2 w.r.t only the drift parameter versus the alternative hypothesis A_2 (not H_2). We call this test the adaptive test. To construct the test statistics of the adaptive tests, we utilize the adaptive maximum likelihood type estimators of both the diffusion and drift parameters and provide three kinds of the adaptive test statistics: likelihood ratio type, Wald type and Rao's score type adaptive test statistics. It is proved that the adaptive test statistics converge in distribution to the chi-squared distribution under the null hypothesis and the adaptive tests are consistent against the alternatives. Moreover, it is shown that the adaptive test statistics converge in distribution to the non-central chi-squared distribution under local alternatives. Some examples are also given and simulation studies show the asymptotic behavior of the three kinds of the adaptive test statistics.

Jump regressions revisited

Mathias Vetter[1]; Fan Yu[2]

[1] Christian-Albrechts-Universität zu Kiel, vetter@math.uni-kiel.de

[2] Christian-Albrechts-Universität zu Kiel, yu@math.uni-kiel.de

Abstract:

A relevant question in the econometrics literature is whether a jump in one stochastic process Z triggers a jump in a related process Y. Starting with the work by Li, Todorov and Tauchen (2017), several papers have discussed this issue, typically in the situation where a jump in Z forces Y to have a jump as well, with the size of the jump in Y given as a function of the simultaneous jump in Z. Asymptotics are then derived in a high-frequency setting, often with the functional relation being linear and based on finite activity jumps in Y and Z.

In this talk, we will discuss more realistic scenarios, including infinite activity jumps and a more classical regression assumption, namely that the jump sizes in Y are not given exactly by a function of the corresponding jump size in Z, but involving additional i.i.d. errors. We will sketch how asymptotical results can be obtained in two different situations.

Inference on jumps in high-frequency order-price models with one-sided noise

Markus Bibinger[1]

[1] Julius-Maximilians-Universität Würzburg, E-mail: markus.bibinger@uni-wuerzburg.de

Abstract: For high-frequency intra-day price processes it is well-known that market microstructure dilutes the underlying dynamics of a semi-martingale model. Different to the classical model for traded prices with additive, centred market microstructure noise, we consider a stochastic boundary model with one-sided noise for best-ask prices from the limit order book. We construct methods to estimate, locate and test for jumps in this model. The different structure of our irregular noise leads us to statistics based on local minima instead of local averages, which have been used for regular noise in the literature. We first discuss inference for a possible jump at some given (stopping) time. We provide a local test and show that we can consistently estimate price jumps. We develop a global test for jumps based on extreme value theory. We establish the asymptotic distribution of a test statistic under the null hypothesis and consistency under the alternative hypothesis. The rate of convergence for local alternatives is faster than in the regular noise model what allows the identification of smaller jumps based on discrete observations. In the process, we establish central limit theorems for spot volatility estimation.

M-estimation based on quasi-processes from discrete samples of Lévy processes

*Yasutaka Shimizu[1]; Hiroshi Shiraishi[2]

[1] Department of Applied Mathematics, Waseda University, E-mail: shimizu@waseda.jp
[2] Department of Mathematics, Keio University, E-mail: shiraishi@math.keio.ac.jp

Abstract:

Let $X = (X_t)_{t\geq 0}$ be a Lévy process starting from $X_0 = u$, which is a random element values in $\mathbb{D}_\infty$ that is a space of càdlàg functions on $\mathbb{R}_+ := [0,\infty)$ with the Skorokhod topology. Given a function $h_\vartheta(x) = h(x,\vartheta) : \mathbb{D}_\infty \times \Theta \to \mathbb{R}$, our interest is an M-estimation for a parameter defined as

$$\vartheta_0 = \arg\min_{\vartheta\in\Theta} \mathbb{E}[h_\vartheta(X)]$$

from discrete observations of $X = (X_t)_{t\geq 0}$.

An example of h_ϑ from insurance and finance is the following functional:

$$h_\vartheta(x) = \int_0^{\tau_\vartheta(x)} e^{-rt} U_\vartheta(x_t)\,\mathrm{d}t, \quad x = (x_t)_{t\geq 0} \in \mathbb{D}_\infty,$$

where $\tau_\vartheta(x) := \inf\{t > 0 \,|\, x_t < \vartheta\}$ and $U_\vartheta : \mathbb{R} \to \mathbb{R}$ for each $\vartheta \in \Theta$. Such a problem typically appears in the 'ruin' or 'default' analysis. The difficulty of this problem is that h_ϑ depends on whole the path of X, and that the random time $\tau_\vartheta(X)$ is usually unobservable in practice. Therefore it is hard to construct an estimator of $\mathbb{E}[h_\vartheta(X)]$ from discrete samples of X.

To overcome the difficulty, we produce "quasi-paths" by shuffling increments of the Lévy process, we call it *quasi-processes*. By shuffling the order of increments randomly and connecting them as step functions, we can produce many quasi-paths of the Lévy process, and we can show that the distribution of the quasi-process converges weakly to the true process in $\mathbb{D}_\infty$-space under a high-frequent sampling scheme. In the talk, we will confirm the convergence result in some numerical experiments.

Using such a resampling technique like the bootstrap, we can estimate the objective functional $\mathbb{E}[h_\vartheta(X)]$ as if we used the Monte Carlo method, and it is available as a contrast function to estimate ϑ_0. We further show that the M-estimator based on these quasi-processes can be consistent and asymptotically normal under suitable conditions. Although we shall give only theoretical results in this talk, an example from dividend strategy in insurance is discussed in an another paper Shimizu and Shiraishi (2021).

References

[1] Shimizu, Y. and Shiraishi, H. (2021). Semiparametric Estimation of Optimal Dividend Barrier for Spectrally Negative Lévy Process, *preprint*.