Kramers' type law for Lévy flights

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1 Abstract

We study the exit problem of solutions of the stochastic differential equation $dX_t^\varepsilon = -U'(X_t^\varepsilon)\,dt + \varepsilon\,dL_t$ from bounded or unbounded intervals which contain the unique asymptotically stable critical point of the deterministic dynamical system $\dot{Y}_t = -U'(Y_t)$. The process L is composed of a standard Brownian motion and a symmetric α-stable Lévy process. Using probabilistic estimates we show that in the small noise limit $\varepsilon \to 0$, the exit time of X^ε from an interval is an exponentially distributed random variable and determine its expected value. Due to the heavy-tail nature of the α-stable component of L, the results differ strongly from the well known case in which the deterministic dynamical system undergoes purely Gaussian perturbations (Kramers' law, Freidlin–Wentzel theory).

We also discuss the physical motivation for this problem which comes from the analysis of Greenland paleoclimatic ice-core data.