

On invariant measures of diffusion processes with unbounded drifts

Vladimir Bogachev

Moscow State University, Russia

The talk gives a survey of recent investigations of invariant measures of diffusion processes with possibly unbounded drifts. The main object in the talk is the weak elliptic equation

$$L^*\mu = 0$$

for measures μ on \mathbb{R}^d , where L is a second order elliptic operator and the above equation is understood in the following weak sense:

$$\int Lf d\mu = 0$$

for all compactly supported smooth functions f . A typical example of L is $\Delta f + (b, \nabla f)$, where b is a vector field. However, b may be quite singular with respect to Lebesgue measure: the only requirement is that b be locally integrable with respect to μ . An important motivation to study such equations is that they include the Kolmogorov equation for stationary distributions of diffusion processes. Major problems concern existence and uniqueness of solutions, existence of densities, and regularity and bounds for densities. In the past decade considerable progress has been achieved in this area. However, there are challenging open problems with very simple formulations. All necessary concepts will be explained in the talk; no special knowledge is assumed, in particular, no acquaintance with diffusion processes is assumed.