Error bounds for high–dimensional Edgeworth expansions for some tests in Multivariate Analysis

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ABSTRACT: Some standard test statistics in multivariate analysis have a common nature, that is, the h–th moment can be written as the ratio of several gamma functions. Let Z be a such statistic. Then the cumulant generating functions of $\log Z$ can be written as the sum of log gamma functions. Using the asymptotic expansion of the log gamma function due to Barnes (see Erdélyi et al. (1953), page 48), the distribution function of $\log Z$ can be expanded in terms of chi–square distribution functions (see Muirhead 1982, page 304). This method can be applied for the (modified) likelihood ratio test statistics testing the equality of covariance matrices of several normal populations, the sphericity, the equality of a covariance matrix to a specified one, and testing specified values for the mean vector and covariance matrix. However, it is known that when the sample sizes are not very large relative to the dimension of the variable, the asymptotic expansions based on chi–square distributions do not give good approximation formulas.
Since all the cumulants of $\log Z$ can be written as the sum of polygamma functions, we can use the Edgeworth expansion of the distribution of $\log Z$. In this paper we give computable upper bounds of the sup–norm of the difference between the exact distribution function and the approximated distribution function by using the asymptotic expansions. We also show some tables of the actual error bounds, which show that the obtained error bounds are sufficiently small for actual use.

Keywords: *Edgeworth expansion*, *computable error bounds*, *equality*, and *sphericity*

References

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