Power Analysis for Structural Equation Modeling with Incomplete Data from an Unknown Population Distribution

Normal-distribution-based likelihood ratio statistic $T_{ML} = (N-1)F_{ML}$ is widely used for power analysis in structural equation modeling (SEM), where F_{ML} is the minimized discrepancy between the sample covariance matrix $\mathbf{S}$ and the structural model $\mathbf{\Sigma}(\boldsymbol{\theta})$. A classical procedure for evaluating power is to compare $(T_{ML}|H_0) \sim \chi^2_{df}$ against $(T_{ML}|H_a) \sim \chi^2_{df}(\delta)$, where the non-centrality parameter δ is computed under a specific alternative population covariance matrix $\mathbf{\Sigma}_a$ (Satorra & Saris, 1985). Comparing χ^2_{df} against $\chi^2_{df}(\delta)$ for power was also used in the procedure proposed by MacCallum, Browne and Sugawara (1996), who suggested to directly specify a value of δ via root mean square error of approximation (RMSEA) instead of specifying an $\mathbf{\Sigma}_a$. Since real data tend to be nonnormally distributed and consequently T_{ML} may not follow a chi-square distribution, Muthén and Muthén (2002) proposed to replace $(T_{ML}|H_a) \sim \chi^2_{df}(\delta)$ by the empirical distribution estimated via simulation. That is, power is estimated by the average rate of $(T_{ML}|H_a) > c_{1-\alpha}$ across replications, where $c_{1-\alpha}$ is the $(1-\alpha)$th quantile of χ^2_{df}. This idea was proposed earlier by Yung and Bentler (1996) and evaluated via bootstrap simulation. Clearly, power provided by these existing procedures can be misleading when $(T_{ML}|H_a) \sim \chi^2_{df}(\delta)$ and especially $(T_{ML}|H_0) \sim \chi^2_{df}$ fail to hold. In practice, these two distribution assumptions may fail to hold with violation of normality, incomplete data, or not a large enough sample size.

For a given statistic T in SEM, we propose to use its empirical distributions under H_0 and H_a for power analysis. In particular, the critical value $c_{1-\alpha}$ is replaced by an estimated $(1-\alpha)$th quantile of T under H_0 obtained via Monte Carlo replications. Denote the estimated quantile as $\hat{c}_{1-\alpha}$, then the power in our proposal is obtained by the average rate of $(T|H_a) > \hat{c}_{1-\alpha}$ across Monte Carlo replications. Since the true distribution of T is consistently estimated by its empirical distribution under either H_0 or H_a, our proposed method will yield consistent power with nonnormal or incomplete data or when the number of variables is large while the sample size is not large enough. However, the power of T in SEM is inversely proportional to the kurtosis of the underlying population distribution. We further propose to use robust methods to deal with nonnormality or heavy tails with real data to increase the power. Using the terminology of power analysis, robust methods effectively reduce the sampling error in estimating the population covariance matrix and consequently increase the effect size for power analysis in SEM.

In our study, in addition to T_{ML}, we also consider a rescaled statistic T_{RML}, and each statistic is evaluated following both the normal-distribution-based maximum likelihood (NML) and robust M-estimation. Results indicate that type I errors are comparable following NML and the M-estimation. However, with non-normally distributed data, especially when the underlying distribution possess heavy tails, statistics following the robust method can be much more powerful than following NML. Existing methods yield reliable power only when the underlying population distribution is normal. Missing data has little effect on our proposed method and there is little difference between the two statistics. But power and type I errors between T_{ML} and T_{RML} differ substantially when they are used following existing methods, especially when data are nonnormally distributed.

References

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