The SIML Estimation of Integrated Covariances and Hedging Coefficients under Micro-market Noise, Round-off Errors and Random Sampling [a]

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[a] This talk is based on unpublished papers with Hiroumi Misaki and Seisho Sato (University of Tokyo) which are available at http://www.e.u-tokyo.ac.jp/cirje/research/dp.

Outline of Presentation

1. Introduction
2. Problems of Micro-market adjustments, Round-off errors and Random Sampling
3. SIML (Separating Information Maximum Likelihood) estimation of Integrated Covariances and Hedging Coefficients
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5. Simulations
6. Concluding Remarks

1 Motivations of Study

1. Recently a considerable interest has been paid on the estimation problem of the integrated volatility by using (ultra) high-frequency financial data. Several statistical methods have been developed by Anderson, T.G., Bollerslev, T. Diebold, F.K. and Labys, P. (2000 JASA), Gloter and Jacod (2001), Ait-Sahalia, Y., P. Mykland and L. Zhang (2005), Zhang, L., P. Mykland and Ait-Sahalia (2005), Hayashi and Yoshida (2005), Barndorff-Nielsen, O., P. Hansen, A. Lunde and N. Shepard (2008, 2011), and Malliavin and Mancino (2009).

2. Our aim is to develop a simple (non-parametric) estimation method for practical applications with micro-market noise. We have proposed to use the SIML (Separating Information Maximum Likelihood) method by Kunitomo and Sato (2008, 2011).

3. We investigate the robustness property of the SIML estimation when we have the micro-market adjustment mechanism, the round-off errors and *Random Sampling* in the process forming the observed prices. The micro-market models including the price adjustments mechanisms have been discussed in the *micro-market literature* in financial economics. We consider the nonlinear price adjustment models while we regard a continuous martingale as the hidden intrinsic value of underlying security including the round-off error models when the high frequency data are randomly sampled.

4. The SIML estimation of the integrated volatility, covariances and the hedging coefficients are asymptotically robust in these situations; that is, they are consistent and asymptotically normal (in the meaningful sense) as the sample size increases under a reasonable set of assumptions. The asymptotic robustness of the SIML method for the underlying continuous stochastic process with micro-market noise in the multivariate non-Gaussian cases.

2 Micro-market adjustments, the Round-off error models and Random Sampling

A General Formulation

We take $p = 2$ and consider $\mathbf{y}_{sf} = (y_s(t_i^s), y_f(t_j^f))'$ such that

$$y_s(t_i^s) = h_s\left(X(t), y_s(t_{i-1}^s), u_s(t_i^s)\ ,\ 0 \le t \le t_i^s\right)\ (i = 1, \cdots, n_s^*)\ ,$$

$$y_f(t_j^f) = h_f\left(X(t), y_f(t_{j-1}^f), u_f(t_j^f)\ ,\ 0 \le t \le t_j^f\right)\ (j = 1, \cdots, n_f^*)$$

and

$$X_t = X_0 + \int_0^t C_x(s) dB_s\ (0 \le t \le 1)\ ,$$

where the (unobservable) continuous martingale process $X(t)$ $(p \times 1)$ generated by Brownian motions $(q \times 1)$ and $u(t_i^s)$ and $u(t_j^f)$ are the micro-market noises.

For the simplicity, we set $p = q = 2$ (the dimensions of observed variables and Brownian Motions, respectively) and assume that

$\mathcal{E}(u_s(t_i^s)) = 0, \mathcal{E}(u_f(t_j^f)) = 0$; $\mathcal{E}(u_s(t_i^s)^2) = \sigma_{ss}^{(u)}$, $\mathcal{E}(u_f(t_j^f)^2) = \sigma_{ff}^{(u)}$,
$\mathcal{E}(u_s(t_i^s)u_f(t_j^f)) = \sigma_{sf}^{(u)}\delta(t_i^s, t_j^f)$; $0 = t_0^s \leq t_1^s \leq \cdots \leq t_{n_s^*}^s$,
$0 = t_0^f \leq t_1^f \leq \cdots \leq t_{n_f^*}^f$, and $h_s(\cdot)$ and $h_f(\cdot)$ are measurable functions.

We want to estimate
(i) the integrated volatilities $\int_0^1 \sigma_{ss}^{(x)}(s)ds$ (and $\int_0^1 \sigma_{ff}^{(x)}(s)ds$),
(ii) the integrated covariance $\int_0^1 \sigma_{sf}^{(x)}(s)ds$
and
(iii) the hedging coefficient $H = \int_0^1 \sigma_{sf}^{(x)}(s)ds / \int_0^1 \sigma_{ff}^{(x)}(s)ds$.

Assumption 2.1 : There exist positive constants $c_{(a)}$ $(a = s, f)$ such that

$$\max_i t_i^{(a)} \longrightarrow 1 \ , \ \frac{n_{(a)}^*}{n} \stackrel{p}{\longrightarrow} c_{(a)}$$

and

$$\mathcal{E}\left[|t_i^{(a)} - t_{i-1}^{(a)}| \right] = O(n^{-1})$$

as $n \to \infty$, where $a = s$ or $a = f$ and $n_{(a)}^*$ are the (random) sample sizes. (n is an index of sample size and we set $c_{(a)} = 1$ without loss of generality.)

Assumption 2.2 : The stochastic process $X(t)$ $(0 \leq t \leq 1)$ is independent of the random sequences t_i^s and t_j^f $(j \geq 1)$.

Example 1 (Equi-distant Sampling) : $t_i^{(a)} - t_{i-1}^{(a)} = 1/n$ ($a = s$ or $a = f$) and $i = 1, \cdots, n$.

Example 2 (Poisson Random Sampling) : $t_i^{(a)}$ ($a = s$ or $a = f$) follows the Poisson Process with λ_n $(= c_{(a)} n)$.

Example 3 (EACD(1,1)) (Engle=Russel (2008), Autoregressive Conditional Duration Models) : Let $\tau_i^{(a)} = t_i^{(a)} - t_{i-1}^{(a)}$ and $\tau_i^{(a)} = \psi_i^{(a)} \epsilon_i^{(a)}$ such that

$$\psi_i^{(a)} = \omega^{(a)} + \alpha^{(a)} \tau_{i-1}^{(a)} + \beta^{(a)} \psi_{i-1}^{(a)}$$

and $\epsilon_i^{(a)}$ are the sequence of i.i.d. exponential random variables with $\alpha^{(a)} > 0, \beta^{(a)} > 0$ and $\omega^{(a)} > 0$.

(i) **Basic Additive Model**

When $p = q = 1$, the basic additive model is represented by the observed log-price $y(t_i^n)$ as

$$y(t_i^n) = X(t_i^n) + u(t_i^n) \ ,$$

where the continuous martingale is given by

$$X_t = X_0 + \int_0^t c_x(s) dB_s \ (0 \leq t \leq 1) \ ,$$

$\Sigma_x(s) = \sigma_x^2(s) = c_x^2(s)$ (the instantaneous volatility) and $u(t_i^n)$ is the micro-market noise.

We want to estimate the integrated volatility $\Sigma_x = \sigma_x^2 = \int_0^1 c_x^2(s)ds$.

There are several important cases of the present formulation for modeling the financial markets with high frequency financial data.

(ii) **A Micro-market price Adjustment model**

We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$. We consider the (linear) micro-market price adjustment model

$$P^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) = g^{(a)} \left[X^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) \right] + u^{(a)}(t_i^{(a)}) \ ,$$

where $X(t)$ (the intrinsic vector of securities at t) and $P^{(a)}(t_i^{(a)})$ are measured in logarithms, the adjustment (constant) coefficient $g^{(a)}$ $(0 < g^{(a)} < 2)$, and $u^{(a)}(t_i^{(a)})$ are i.i.d. sequence of noises with $\mathcal{E}[u^{(a)}(t_i^{(a)})] = 0$ and $\mathcal{E}[u^{(a)}(t_i^{(a)})^2] = \sigma_{aa}^{(u)}$.

(iii) **The Round-off-error model**

We assume that

$$P^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) = g_\eta^{(a)}\left[X^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) + u^{(a)}(t_i^{(a)})\right] \ ,$$

where $u^{(a)}(t_i^{(a)})$ is an i.i.d. noise with $\mathcal{E}[u^{(a)}(t_i^{(a)})] = 0$,$\mathcal{E}[u(t_i^{(a)})^2] = \sigma_{aa}^{(u)}$ and the nonlinear function

$$g_\eta^{(a)}(x) = \eta\left[\frac{x}{\eta}\right] \ ,$$

where $g_\eta(y)$ is the integer part of y and $[y]$ is the largest integer being less than y and η is a small positive constant.

This model corresponds to the micro-market model with the restriction of the minimum price change and η is the parameter of minimum price change. We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$.

(iv) **Nonlinear Micro-market price Adjustment models**

We take a non-linear version with

$$g(x) = g_1 x \mathrm{I}(x \geq 0) + g_2 x \mathrm{I}(x < 0) \ ,$$

where g_i $(i = 1, 2)$ are some constants and $\mathrm{I}(\cdot)$ is the indicator function. (This has been called the SSAR (simultaneous switching autoregressive) model, whic have been investigated by Sato and Kunitomo (1996) and Kunitomo and Sato (1999).) A set of sufficient conditions for the geometric ergodicity of the price process is given by $g_1 > 0$, $g_2 > 0$, $(1 - g_1)(1 - g_2) < 1$.

More generally, we consider the model

$$P(t_i^{(a)}) - P(t_{i-1}^{(a)}) = g\left[X(t_i^{(a)}) - P(t_{i-1}^{(a)})\right] + u(t_{i-1}^{(a)}) \ ,$$

where $u(t_i^{(a)})$ is an i.i.d. sequence of noise with $\mathcal{E}[u(t_i^{(a)})] = 0$ and $\mathcal{E}[u(t_i^{(a)})^2] = \sigma_{aa}^u$. We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$.

3 SIML: Separating Information Maximum Likelihood) estimation

Let y_{ij} be the $i-$th observation of the $j-$th (log-) price at t_i^n in the equidistant case, that is, for $j = 1, \cdots, p; i = 1, \cdots, n$
$0 = t_0^n \leq t_1^n \leq \cdots \leq t_n^n = 1$. In the general case, this paper uses the refreshing-time method.

We set $\mathbf{y}_i = (y_{i1}, \cdots, y_{ip})^{'}$ be a $p \times 1$ vector and $\mathbf{Y}_n = (\mathbf{y}_i^{'})$ be an $n \times p$ matrix of observations. The underlying continuous process $\mathbf{x}_i$ at t_i^n $(i = 1, \cdots, n)$ is not necessarily the same as the observed prices and let $\mathbf{v}_i^{'} = (v_{i1}, \cdots, v_{ip})$ be the vector of the additive micro-market noise at t_i^n, which is independent of $\mathbf{x}_i$. Then we have

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i$$

where $\mathbf{v}_i$ are a sequence of independent random variables with $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and $\mathcal{E}(\mathbf{v}_i\mathbf{v}_i^{'}) = \mathbf{\Sigma}_v$. We sometimes refer to the equi-distant (one-dimension) case with $h_n = t_i^n - t_{i-1}^n = 1/n$ $(i = 1, \cdots, n)$ and $p = q = 1$.

We assume that

$$X(t) = X(0) + \int_0^t C_x(s) dB_s \ \ (0 \leq t \leq 1),$$

and $x_i = X(t_i^n)$, where B_s is the standard Brownian motion, $C_x(s)$ is progressively measurable in $[0, s] \times \mathcal{F}_s$ and predictable, and

$$\Sigma_x = \int_0^1 C_x(s) C_x^{'}(s) ds \ .$$

Three different situations:
(i) When the coefficient matrix is constant, (i.e. $C_x(s)C_x(s)^{'} = \Sigma_x$), we call the simple case.
(ii) When the coefficient matrix is time-varying, but it is a deterministic function of time ($C_x(s)$), we call the deterministic time-varying case.
(iii) When the coefficient matrix is time-varying and it is a stochastic function of time ($C_x(s)$) and Σ_x is random, we call the stochastic case.

The Basic Case when $p = q = 1$

We consider the situation when $\mathbf{x}_i$ and $\mathbf{v}_i$ $(i = 1, \cdots, n)$ are independent, and $\mathbf{v}_i$ are independently and normally distributed as $N_p(\mathbf{0}, \sigma_v^2)$. Given the initial condition y_0,

$$\mathbf{y}_n \sim N_n \left(\mathbf{1}_n \cdot y_0, \mathbf{I}_n \otimes \sigma_v^2 + \mathbf{C}_n \mathbf{C}_n^{'} \otimes h_n \sigma_x^2 \right) ,$$

where $\mathbf{1}_n^{'} = (1, \cdots, 1)$, $h_n = 1/n$ $(= t_i^n - t_{i-1}^n)$ and

$$\mathbf{C}_n = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ 1 & \cdots & 1 & 1 & 0 \\ 1 & \cdots & 1 & 1 & 1 \end{pmatrix} .$$

$$\mathbf{C}_n^{-1} \mathbf{C}_n^{'-1} = \mathbf{P}_n \mathbf{D}_n \mathbf{P}_n^{'} = 2\mathbf{I}_n - 2\mathbf{A}_n ,$$

where $\mathbf{D}_n$ is a diagonal matrix with $d_k = 2 \left[1 - \cos(\pi(\frac{2k-1}{2n+1})) \right]$, and

$\mathbf{P}_n = (p_{jk})$, $p_{jk} = \sqrt{\frac{2}{n+\frac{1}{2}}} \cos\left[\pi(\frac{2k-1}{2n+1})(j-\frac{1}{2})\right]$.

We transform $\mathbf{y}_n$ to $\mathbf{z}_n$ $(= (z_k))$ by $\mathbf{z}_n = h_n^{-1/2}\mathbf{P}_n^{'}\mathbf{C}_n^{-1}(\mathbf{y}_n - \bar{\mathbf{y}}_0)$ and $\bar{\mathbf{y}}_0 = \mathbf{1}_n \cdot y_0$. The likelihood function under the Gaussian noise when $p = q = 1$ is given by

$$L_n^*(\boldsymbol{\theta}) = \left(\frac{1}{\sqrt{2\pi}}\right)^{np} \prod_{k=1}^{n} |a_{kn}\sigma_v^2 + \sigma_x^2|^{-1/2} e^{\left\{-\frac{1}{2}z_k^2\left(a_{kn}\sigma_v^2 + \sigma_x^2\right)^{-1}\right\}}$$

where $a_{kn} = 4n\sin^2\left[\frac{\pi}{2}\left(\frac{2k-1}{2n+1}\right)\right]$. The maximum likelihood (ML) estimator can be defined as the solution of maximizing

$$L_n(\boldsymbol{\theta}) = \sum_{k=1}^{n} \log|a_{kn}\sigma_v^2 + \sigma_x^2|^{-1/2} - \frac{1}{2}\sum_{k=1}^{n} z_k^2[a_{kn}\sigma_v^2 + \sigma_x^2]^{-1} .$$

The ML estimator of unknown parameters is a rather complicated function of all observations and each a_{kn} terms depend on k as well as n. Let denote $a_{k_n,n}$ and then we can evaluate that $a_{k_n,n} \to 0$ as $n \to \infty$ when $k_n = O(n^\alpha)$ $(0 < \alpha < \frac{1}{2})$ since $\sin x \sim x$ as $x \to 0$. Also $a_{n+1-l_n,n} = O(n)$

when $l_n = O(n^{\beta})$ $(0 < \beta < 1)$.
When k_n is small, we expect that $a_{k_n,n}$ is small and we approximate $2 \times L_n(\boldsymbol{\theta})$ by

$$L_n^{(1)}(\boldsymbol{\theta}) = -m \log |\sigma_x^2| - \sum_{k=1}^{m} z_k^2 {\sigma_x^{-2}}$$

and then the SIML estimator is defined by

$$\hat{\boldsymbol{\Sigma}}_x = \frac{1}{m_n} \sum_{k=1}^{m_n} z_k^2 \ .$$

The number of terms m_n should be dependent on n and we need the requirements that $m_n = O(n^{\alpha})$ $(0 < \alpha < \frac{1}{2})$. We can alternatively write

$$\hat{\Sigma}_x = \sum_{i,j=1}^{n} c_{ij}(y_i - y_{i-1})(y_j - y_{j-1}) \ .$$

In the more general caese when $p \geq 1$

$$\hat{\mathbf{\Sigma}}_x = \frac{1}{m_n} \sum_{k=1}^{m_n} \mathbf{z}_k \mathbf{z}_k^{'} \ .$$

4 Asymptotic Properties of the SIML and Asymptotic Robustness

4.1 A summary of Asymptotic Properties ($p = q = 1$)

We summarize the asymptotic properties of the SIML estimator when the sample size n is large. Kunitomo and Sato [2008, 2011] have investigated the problem and have shown that the SIML estimator is consistent and it has the asymptotic normality under a set regularity conditions when $p = q = 1$.
As $n \longrightarrow \infty$

$$\hat{\sigma}_x^2 - \sigma_x^2 \xrightarrow{p} 0$$

with $m_n = n^{\alpha}$ $(0 < \alpha < 1/2)$ and

$$\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right] \xrightarrow{d} N\left(0, 2[\sigma_x^2]^2\right)$$

with $m_n^5/n^2 \rightarrow 0$.

Although the SIML estimation was introduced under the Gaussian process and the standard model, it has reasonable finite sample properties as well as asymptotic properties under some volatility models and the non-Gaussian processes with

$$\mathcal{E}\left[(x_i - x_{i-1})^2 | \mathcal{F}_{n,i-1}\right] = \int_{t_{i-1}}^{t_i} C_x^2(s) ds \ .$$

As $n \longrightarrow \infty$, under a set of regularity conditions, the asymptotic distribution of the SIML estimator of the integrated variance can be summarized as

$$\sqrt{m_n}\left[\hat{\sigma}_{xx} - \sigma_{xx}\right] \xrightarrow{d} N\left[0, V_{xx}\right] \ ,$$

provided that we have the convergence of the asymptotic variance

$$V_{xx} = 2\left[\int_0^1 C_x^4(s) ds\right]$$

and it is a positive constant when $m_n^5/n^2 \rightarrow 0$ (as $n \rightarrow \infty$).

Remark : When V_{xx} is a random variable, the convergence is in the sense of *stable convergence*.

When $p = q = 2$, as $n \longrightarrow \infty$, under a set of regularity conditions, the asymptotic distribution of the SIML estimator of integrated covariance can be summarized as

$$\sqrt{m_n}\left[\hat{\sigma}_{sf} - \sigma_{sf}\right] \xrightarrow{d} N\left[0, V_{sf}\right] \ ,$$

provided that we have the convergence of the asymptotic variance

$$V_{sf} = \int_0^1 \left[\sigma_{ss}^{(x)}(s)\sigma_{ff}^{(x)}(s) + \sigma_{sf}^2(s)\right] ds \ .$$

4.2 Asymptotic Robustness under Micro-market adjustments and the Round-off error models ($p = q = 1$)

(i) **A Micro-market price Adjustment model**

Theorem 3-3 (Sato-Kunitomo) : Assume $0 < g < 2$ and Define the SIML estimator of the realized volatility of $X(t)$ with $m_n = n^\alpha$ $(0 < \alpha < 0.4)$. Then the asymptotic distribution of $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically $(m_n, n \to \infty)$ equivalent to the limiting distributions under the standard additive (i.e. the signal-plus-noise) models.

(ii) **The Round-off-error model**

Theorem 3.4 (Sato-Kunitomo) : Set $\eta = \eta_n$ depending on n with $\eta_n \sqrt{n} = O(1)$. Define the SIML estimator of the realized volatility of $X(t)$ with $m_n = n^{\alpha}$ $(0 < \alpha < 0.4)$. The limiting random variable of the normalized estimator $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically $(m_n, n \to \infty)$ equivalent to the limiting distributions in the standard models.

(iii) **Nonlinear Micro-market price Adjustment models**

Theorem 3-5 (Sato-Kunitomo) : For the non-linear time series process $V(t_i^n)$ we assume that there exist functions $\rho_1(\,\cdot\,)$ and $\rho_2(\,\cdot,\cdot\,)$ such that $\mathrm{Cov}[V(t_i^n),V(t_j^n)] = c_1\rho_1(|i-j|)$, where c_1 is a (positive) constant and $\sum_{s=0}^{\infty}\rho_1(s) < \infty$ and
$\mathrm{Cov}\left[V(t_i^n)V(t_{i'}^n),V(t_j^n)V(t_{j'}^n))\right] = c_2\rho_2(|i-i^{'}|,|j-j^{'}|)$, where c_2 is a (positive) constant and $\sum_{s,s'=0}^{\infty}\rho_2(s,s^{'}) < \infty$.
Define the SIML estimator of the realized volatility of $P(t_i^n)$ with $m_n = n^{\alpha}$ $(0<\alpha<0.4)$. Then the asymptotic distribution of $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically (as $m_n, n \to \infty$) equivalent to the limiting distributions under the standard models.

5 Simulations

5-1 Simulations in Sato-Kunitomo (2011) : Equi-distant Case ($p = q = 1$)

The the volatility function is given by

$$\sigma_x^2(s) = \sigma(0)^2 \left[a_0 + a_1 s + a_2 s^2\right],$$

where a_i $(i = 0, 1, 2)$ are constants and $\sigma_x(s)^2 > 0$ for $s \in [0, 1]$. It is a typical time varying (but deterministic) case and

$$\sigma_x^2 = \int_0^1 \sigma_x(s)^2 ds = \sigma_x(0)^2 \left[a_0 + \frac{a_1}{2} + \frac{a_2}{3}\right] .$$

We have examined several models

$$
\begin{aligned}
&\text{Model 1} \qquad h_1(x,y,u) = y + g(x-y) + u \ (g : \text{a const}) \ , \\
&\text{Model 2} \qquad h_2(x,y,u) = y + g_\eta(x-y+u) \ \ (g_\eta(\cdot) \text{ is } (3.8)) \ , \\
&\text{Model 3} \qquad h_3(x,y,u) = y + g_\eta(x-y) + u \ \ (g_\eta(\cdot) \text{ is } (3.8)) \ , \\
&\text{Model 4} \qquad h_4(x,y,u) = y + u + \begin{cases} g_1(x-y) & \text{if } y \geq 0 \ (g_1 : \text{a const}) \\ g_2(x-y) & \text{if } y < 0 \ (g_2 : \text{a const}) \end{cases} \ , \\
&\text{Model 5} \qquad h_5(x,y,u) = y + \left[g_1 + g_2 \exp(-\gamma|x-y|^2)\right](x-y) \ (g_1, g_2 : \text{const}) \\
&\text{Model 6} \qquad h_6(x,y,u) = y + g_1 \sin\left(g_2(x-y)\right) \ (g_1, g_2 : \text{const}) \ , \\
&\text{Model 7} \qquad h_7(x,y,u) = y + h_2 \ o \ h_4 \ o \ h_1(x,y,u) \ ,
\end{aligned}
$$

respectively.

B-1 : Estimation of integrated volatility (Model-1)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E}-04, g = 0.2)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.01E+00	2.33E+00	1.04E+00
SD	1.97E-01	2.32E-02	6.58E-02
MSE	3.89E-02	1.78E+00	6.00E-03

B-2 : Estimation of integrated volatility (Model-1)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E}+00, g = 0.2)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	9.96E-01	1.11E-01	9.71E-01
SD	1.93E-01	2.35E-03	6.30E-02
MSE	3.74E-02	7.90E-01	4.80E-03

B-3 : Estimation of integrated volatility (Model-1)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E} + 00, g = 1.5)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.00E+00	3.00E+00	1.01E+00
SD	1.94E-01	4.03E-02	6.55E-02
MSE	3.78E-02	4.00E+00	4.34E-03

B-4 : Estimation of integrated volatility (Model-1)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E} - 05, g = 1.0)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	9.88E-01	1.40E+00	9.97E-01
SD	1.99E-01	1.40E-02	6.53E-02
MSE	3.97E-02	1.60E-01	4.27E-03

B-5 : Estimation of integrated volatility (Model-1)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E} - 06, g = 0.01)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	8.40E-01	2.51E-02	2.48E-01
SD	1.66E-01	5.41E-04	2.76E-02
MSE	5.31E-02	9.50E-01	5.66E-01

B-6 : Estimation of integrated volatility (Model-2)
$(a_0 = 7, a_1 = -12, a_2 = 6; \sigma_u^2 = 2.00\mathrm{E} - 02, \eta = 0.5)$

n=20000	σ_x^2	H-vol	RK
true-val	4.50E+01	4.50E+01	4.50E+01
mean	4.60E+01	1.37E+02	5.36E+01
SD	1.05E+01	6.19E+00	3.65E+00
MSE	1.11E+02	8.46E+03	8.68E+01

B-7 : Estimation of integrated volatility (Model-3)
$(a_0 = 7, a_1 = -12, a_2 = 6; \sigma_u^2 = 1.00\text{E} - 02, \eta = 0.5)$

n=20000	σ_x^2	H-vol	RK
true-val	4.50E+01	4.50E+01	4.50E+01
mean	4.54E+01	3.95E+02	6.19E+01
SD	1.05E+01	6.69E+00	4.07E+00
MSE	1.10E+02	1.22E+05	3.02E+02

B-8 : Estimation of integrated volatility (Model-3)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\text{E} + 00, \eta = 0.005)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.00E+00	6.85E-01	9.97E-01
SD	1.94E-01	8.66E-03	6.21E-02
MSE	3.77E-02	9.92E-02	3.87E-03

B-9 : Estimation of integrated volatility (Model-4)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E} + 00, g_1 = 0.2, g_2 = 5)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.01E+00	2.22E+00	1.01E+00
SD	1.93E-01	6.46E-02	6.25E-02
MSE	3.71E-02	1.49E+00	3.93E-03

B-10 : Estimation of integrated volatility (Model-4)
$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E} - 03, g_1 = 0.2, g_2 = 5)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.02E+00	6.65E+01	1.11E+00
SD	1.94E-01	1.66E+00	7.46E-02
MSE	3.79E-02	4.30E+03	1.85E-02

B-11 : Estimation of integrated volatility (Model-5)

$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E}+00, g_1 = 1.9, g_2 = -1.7, \gamma = 10000)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	9.99E-01	6.39E+00	1.00E+00
SD	1.92E-01	3.66E-01	6.53E-02
MSE	3.68E-02	2.91E+01	4.26E-03

B-12 : Estimation of integrated volatility (Model-6)

$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\mathrm{E}+00, \sin(z * 0.1))$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.00E+00	5.26E-02	8.32E-01
SD	2.14E-01	2.23E-03	6.79E-02
MSE	4.59E-02	8.97E-01	3.27E-02

B-13 : Estimation of integrated volatility (Model-6)

$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\text{E} + 00, 0.01 * \sin(z * 100))$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	7.67E-01	4.49E-01	7.75E-01
SD	1.79E-01	3.78E-03	6.05E-02
MSE	8.64E-02	3.03E-01	5.41E-02

B-14 : Estimation of integrated volatility (Model-7)

$(a_0 = 1, a_1 = 0, a_2 = 0; \sigma_u^2 = 1.00\text{E} - 04, g_1 = 0.2, g_2 = 5; g = 0.01; \eta = 0.01)$

n=20000	σ_x^2	H-vol	RK
true-val	1.00E+00	1.00E+00	1.00E+00
mean	1.18E+00	3.62E+00	1.81E+00
SD	2.30E-01	1.04E-01	1.16E-01
MSE	8.36E-02	6.85E+00	6.69E-01

5-2 Simulations in Kunitomo-Misaki (2013) : Random Sampling Cases

(i) Basic Simulations (p=q=1)

(ii) Extended Simulations (p=q=2)

3-1 : Estimation of integrated volatility : Case 1 ($a_0 = 1, a_1 = a_2 = 0$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.22E-04	2.07E-04	2.05E-04	2.04E-04	2.05E-04
		6.54E-05	4.52E-05	6.44E-05	8.34E-05	9.48E-05	1.31E-04
$\hat{\sigma}_v^2$	2.00E-06	2.03E-06	1.04E-07	9.82E-07	1.92E-06	2.18E-06	3.00E-06
		1.43E-07	6.00E-09	8.43E-08	2.19E-07	3.19E-07	8.30E-07
HI		7.39E-03	7.04E-03	4.74E-03	2.48E-03	1.40E-03	4.40E-04
		3.45E-04	3.35E-04	2.61E-04	1.79E-04	1.34E-04	8.46E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.07E-04	2.07E-04	2.06E-04
		4.12E-05	4.14E-05	6.55E-05	8.45E-05	9.66E-05	1.31E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.40E-07	2.03E-06	2.09E-06	2.18E-06	3.00E-06
		5.54E-08	3.08E-08	1.41E-07	2.33E-07	3.19E-07	8.38E-07
HI		7.22E-02	4.57E-02	7.41E-03	2.60E-03	1.40E-03	4.40E-04
		1.06E-03	7.80E-04	2.96E-04	1.83E-04	1.35E-04	8.63E-05

3-2 : Estimation of integrated volatility: Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	1.67E-04	1.72E-04	1.89E-04	1.74E-04	1.71E-04	1.71E-04	1.72E-04
		5.52E-05	3.88E-05	5.44E-05	7.04E-05	8.01E-05	1.10E-04
$\hat{\sigma}_v^2$	2.00E-06	2.02E-06	1.04E-07	9.78E-07	1.90E-06	2.14E-06	2.83E-06
		1.43E-07	5.99E-09	8.40E-08	2.18E-07	3.15E-07	7.85E-07
HI		7.36E-03	7.01E-03	4.71E-03	2.44E-03	1.36E-03	4.06E-04
		3.44E-04	3.34E-04	2.60E-04	1.77E-04	1.32E-04	7.92E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	1.67E-04	1.72E-04	1.73E-04	1.74E-04	1.74E-04	1.73E-04	1.73E-04
		3.50E-05	3.52E-05	5.56E-05	7.15E-05	8.16E-05	1.11E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.39E-07	2.03E-06	2.08E-06	2.15E-06	2.84E-06
		5.54E-08	3.08E-08	1.41E-07	2.32E-07	3.14E-07	7.92E-07
HI		7.22E-02	4.57E-02	7.37E-03	2.57E-03	1.36E-03	4.07E-04
		1.06E-03	7.80E-04	2.95E-04	1.81E-04	1.33E-04	8.06E-05

3-3 : Estimation of integrated volatility: Case 3 (Stochastic Volatility)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.08E-04	2.23E-04	2.09E-04	2.07E-04	2.08E-04	2.11E-04
		6.94E-05	4.65E-05	6.78E-05	8.91E-05	1.07E-04	1.43E-04
$\hat{\sigma}_v^2$	2.00E-06	2.02E-06	1.04E-07	9.81E-07	1.93E-06	2.16E-06	3.03E-06
		1.42E-07	5.91E-09	8.50E-08	2.19E-07	3.19E-07	8.44E-07
HI		7.39E-03	7.04E-03	4.74E-03	2.48E-03	1.39E-03	4.45E-04
		3.49E-04	3.37E-04	2.52E-04	1.77E-04	1.33E-04	8.72E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04	2.05E-04
		4.09E-05	4.10E-05	6.36E-05	8.17E-05	9.63E-05	1.29E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.40E-07	2.02E-06	2.10E-06	2.19E-06	2.95E-06
		5.53E-08	3.04E-08	1.41E-07	2.27E-07	3.31E-07	8.17E-07
HI		7.22E-02	4.57E-02	7.40E-03	2.60E-03	1.40E-03	4.36E-04
		1.08E-03	7.96E-04	2.93E-04	1.79E-04	1.35E-04	8.61E-05

5-1 : Estimation of integrated volatility: Case 4 (Autoregressive Conditional Duration)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.03E-04	2.23E-04	2.06E-04	2.04E-04	2.03E-04	2.05E-04
		6.75E-05	4.50E-05	6.62E-05	8.41E-05	9.70E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.03E-06	1.04E-07	9.75E-07	1.89E-06	2.17E-06	3.01E-06
		1.44E-07	8.05E-09	9.60E-08	2.18E-07	3.12E-07	8.29E-07
HI		7.41E-03	7.05E-03	4.70E-03	2.46E-03	1.39E-03	4.41E-04
		5.22E-04	4.85E-04	3.01E-04	1.78E-04	1.32E-04	8.47E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.06E-04	2.05E-04	2.04E-04	2.05E-04	2.07E-04
		4.14E-05	4.12E-05	6.49E-05	8.31E-05	9.73E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.31E-07	2.03E-06	2.10E-06	2.19E-06	3.00E-06
		5.56E-08	3.35E-08	1.41E-07	2.28E-07	3.18E-07	8.39E-07
HI		7.22E-02	4.52E-02	7.40E-03	2.61E-03	1.40E-03	4.40E-04
		1.67E-03	9.32E-04	2.95E-04	1.78E-04	1.34E-04	8.67E-05

5-2 : Estimation of integrated volatility: Case 5 ($g = 0.2$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.35E-04	4.22E-04	2.40E-04	2.16E-04	2.10E-04	2.08E-04
		7.42E-05	9.28E-05	7.44E-05	8.85E-05	9.76E-05	1.35E-04
$\hat{\sigma}_v^2$	2.00E-06	6.42E-07	5.67E-08	5.78E-07	1.73E-06	3.21E-06	6.31E-06
		4.58E-08	2.79E-09	4.75E-08	2.03E-07	4.79E-07	1.75E-06
HI		4.01E-03	3.98E-03	3.66E-03	3.07E-03	2.42E-03	8.33E-04
		1.66E-04	1.69E-04	1.89E-04	2.13E-04	2.26E-04	1.71E-04
18000	**True**	**Raw**	**1 sec.**	**10 sec.**	**30 sec.**	**60 sec.**	**300 sec.**
$\hat{\sigma}_x^2$	2.00E-04	2.27E-04	2.30E-04	2.10E-04	2.11E-04	2.10E-04	2.13E-04
		4.57E-05	4.60E-05	6.63E-05	8.60E-05	9.84E-05	1.35E-04
$\hat{\sigma}_v^2$	2.00E-06	6.28E-07	5.63E-07	4.32E-06	5.58E-06	5.72E-06	6.56E-06
		1.73E-08	1.74E-08	3.08E-07	6.24E-07	8.39E-07	1.83E-06
HI		4.00E-02	3.63E-02	1.74E-02	6.82E-03	3.51E-03	8.59E-04
		5.21E-04	5.86E-04	7.00E-04	4.85E-04	3.44E-04	1.79E-04

5-3 : Estimation of integrated volatility: Case 6 ($\eta = 0.001$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.22E-04	2.06E-04	2.05E-04	2.04E-04	2.05E-04
		6.46E-05	4.55E-05	6.36E-05	8.45E-05	9.54E-05	1.34E-04
$\hat{\sigma}_v^2$	2.00E-06	2.11E-06	1.08E-07	1.02E-06	1.99E-06	2.25E-06	3.09E-06
		1.51E-07	6.26E-09	8.79E-08	2.29E-07	3.27E-07	8.52E-07
HI		7.68E-03	7.32E-03	4.93E-03	2.57E-03	1.44E-03	4.50E-04
		3.58E-04	3.47E-04	2.70E-04	1.86E-04	1.37E-04	8.62E-05

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04
		4.08E-05	4.10E-05	6.53E-05	8.48E-05	9.73E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.09E-06	9.78E-07	2.11E-06	2.17E-06	2.26E-06	3.10E-06
		5.76E-08	3.20E-08	1.47E-07	2.41E-07	3.29E-07	8.58E-07
HI		7.52E-02	4.76E-02	7.71E-03	2.70E-03	1.45E-03	4.51E-04
		1.10E-03	8.10E-04	3.06E-04	1.90E-04	1.41E-04	8.84E-05

5-4 : Estimation of integrated volatility: Case 7 ($\eta = 0.001$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.23E-04	2.07E-04	2.05E-04	2.04E-04	2.05E-04
		6.46E-05	4.58E-05	6.36E-05	8.47E-05	9.57E-05	1.34E-04
$\hat{\sigma}_v^2$	2.00E-06	2.11E-06	1.09E-07	1.02E-06	1.99E-06	2.26E-06	3.09E-06
		1.48E-07	6.22E-09	8.75E-08	2.30E-07	3.30E-07	8.48E-07
HI		7.69E-03	7.33E-03	4.93E-03	2.57E-03	1.44E-03	4.50E-04
		3.53E-04	3.43E-04	2.69E-04	1.86E-04	1.38E-04	8.60E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04
		4.08E-05	4.10E-05	6.55E-05	8.51E-05	9.75E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.09E-06	9.78E-07	2.11E-06	2.17E-06	2.26E-06	3.09E-06
		5.73E-08	3.20E-08	1.47E-07	2.38E-07	3.30E-07	8.53E-07
HI		7.52E-02	4.76E-02	7.71E-03	2.70E-03	1.45E-03	4.51E-04
		1.10E-03	8.09E-04	3.07E-04	1.89E-04	1.41E-04	8.79E-05

5-5 : Estimation of integrated volatility: Case 8 ($g_1 = 0.2, g_2 = 5$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.28E-04	3.12E-04	2.32E-04	2.23E-04	2.22E-04	2.25E-04
		7.27E-05	6.87E-05	7.29E-05	9.22E-05	1.05E-04	1.46E-04
$\hat{\sigma}_v^2$	2.00E-06	2.46E-06	1.70E-07	1.69E-06	4.29E-06	5.93E-06	7.08E-06
		2.11E-07	1.46E-08	1.79E-07	5.65E-07	9.76E-07	2.26E-06
HI		1.21E-02	1.17E-02	9.32E-03	6.06E-03	3.72E-03	9.28E-04
		9.54E-04	9.43E-04	8.27E-04	6.15E-04	4.61E-04	2.36E-04
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.24E-04	2.27E-04	2.23E-04	2.25E-04	2.23E-04	2.27E-04
		4.62E-05	4.69E-05	7.21E-05	9.28E-05	1.05E-04	1.45E-04
$\hat{\sigma}_v^2$	2.00E-06	2.35E-06	1.60E-06	5.85E-06	5.88E-06	5.97E-06	6.82E-06
		7.32E-08	6.13E-08	4.46E-07	7.24E-07	9.81E-07	2.14E-06
HI		1.16E-01	8.90E-02	2.11E-02	7.15E-03	3.67E-03	8.98E-04
		3.01E-03	2.55E-03	1.09E-03	6.43E-04	4.56E-04	2.21E-04

3-1 : Estimation of hedging coefficent: Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.75E-05	7.33E-06	4.30E-05	6.34E-05	8.03E-05	9.53E-05
		1.78E-04	5.34E-05	1.24E-04	1.19E-04	9.33E-05	6.15E-05
HY	1.00E-04	1.05E-04	(1.25E-04)				
RCV-RV	5.00E-01	9.08E-03	1.04E-03	8.98E-03	2.55E-02	5.77E-02	2.20E-01
		2.41E-02	7.57E-03	2.60E-02	4.83E-02	6.68E-02	1.39E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.21E-02	4.24E-02	7.59E-02	2.49E-01
		1.70E-02	1.78E-02	2.64E-02	5.06E-02	9.11E-02	3.04E-01
HY-SIML	5.00E-01	5.81E-01	4.87E-01	5.71E-01	6.31E-01	6.86E-01	8.72E-01
		7.54E-01	5.98E-01	7.42E-01	8.74E-01	1.06E+00	1.75E+00
SIML-SIML	5.00E-01	4.97E-01	4.51E-01	4.91E-01	5.01E-01	5.17E-01	5.05E-01
		2.42E-01	1.29E-01	2.05E-01	2.72E-01	3.30E-01	5.11E-01
RCV	1.00E-04	6.67E-05	4.92E-06	3.73E-05	6.84E-05	8.35E-05	9.76E-05
		8.61E-06	2.17E-06	6.10E-06	9.70E-06	1.37E-05	3.01E-05
HY	1.00E-04	1.00E-04	(1.11E-05)				
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	2.45E-01	1.83E-02	1.52E-01	3.08E-01	3.96E-01	4.82E-01
		2.96E-02	8.08E-03	2.40E-02	3.97E-02	5.52E-02	1.16E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.09E-01	4.51E-01	4.77E-01	5.12E-01
		3.76E-02	3.81E-02	4.17E-02	4.62E-02	5.22E-02	1.06E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.10E-01	6.65E-01	9.02E-01
		2.10E-01	1.18E-01	2.01E-01	2.98E-01	4.31E-01	1.21E+00
SIML-SIML	5.00E-01	5.10E-01	5.01E-01	5.11E-01	5.13E-01	5.29E-01	5.24E-01
		2.39E-01	1.23E-01	2.03E-01	2.71E-01	3.36E-01	5.02E-01

3-2 : Estimation of hedging coeffcient: Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	8.27E-05	4.15E-05	1.02E-04	1.02E-04	1.03E-04	1.02E-04
		5.44E-04	3.83E-04	2.10E-04	1.27E-04	9.32E-05	6.32E-05
HY	1.00E-04	9.94E-05	(3.86E-04)				
RCV-RV	5.00E-01	1.14E-03	9.13E-04	1.38E-02	3.91E-02	7.36E-02	2.31E-01
		7.53E-03	8.39E-03	2.83E-02	4.87E-02	6.60E-02	1.37E-01
HY-RV	5.00E-01	1.38E-03	2.18E-03	1.35E-02	3.78E-02	7.09E-02	2.32E-01
		5.35E-03	8.45E-03	5.23E-02	1.49E-01	2.78E-01	9.25E-01
HY-SIML	5.00E-01	5.11E-01	5.07E-01	5.20E-01	5.31E-01	5.70E-01	9.00E-01
		2.00E+00	1.99E+00	2.24E+00	2.52E+00	2.89E+00	5.28E+00
SIML-SIML	5.00E-01	4.91E-01	4.86E-01	4.99E-01	5.12E-01	5.18E-01	5.19E-01
		1.44E-01	1.30E-01	2.02E-01	2.68E-01	3.17E-01	5.16E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.67E-05	3.68E-05	9.00E-05	9.67E-05	9.82E-05	1.00E-04
		6.77E-06	5.28E-06	6.86E-06	9.94E-06	1.37E-05	2.98E-05
HY	1.00E-04	9.99E-05	(6.36E-06)				
RCV-RV	5.00E-01	7.25E-02	5.62E-02	3.31E-01	4.32E-01	4.63E-01	4.95E-01
		7.34E-03	8.04E-03	2.27E-02	3.70E-02	5.18E-02	1.18E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.68E-01	4.47E-01	4.74E-01	5.11E-01
		6.90E-03	9.59E-03	2.45E-02	3.62E-02	4.67E-02	1.00E-01
HY-SIML	5.00E-01	5.22E-01	5.23E-01	5.57E-01	5.93E-01	6.39E-01	8.14E-01
		1.13E-01	1.13E-01	1.98E-01	2.98E-01	4.04E-01	9.23E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.29E-01	5.33E-01	5.40E-01
		1.40E-01	1.25E-01	1.99E-01	2.63E-01	3.12E-01	4.85E-01

3-3 : Estimation of hedging coefficient: Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	7.60E-03	9.30E-04	7.71E-03	2.12E-02	4.90E-02	1.98E-01
		2.40E-02	7.57E-03	2.61E-02	4.85E-02	6.73E-02	1.40E-01
HY-RV	5.00E-01	1.20E-02	1.26E-02	1.87E-02	3.61E-02	6.53E-02	2.26E-01
		1.69E-02	1.77E-02	2.63E-02	5.08E-02	9.24E-02	3.24E-01
HY-SIML	5.00E-01	5.83E-01	4.79E-01	5.72E-01	6.37E-01	6.96E-01	8.68E-01
		8.82E-01	6.95E-01	8.66E-01	1.02E+00	1.23E+00	1.98E+00
SIML-SIML	5.00E-01	4.94E-01	4.43E-01	4.88E-01	4.98E-01	5.14E-01	5.02E-01
		2.44E-01	1.31E-01	2.08E-01	2.74E-01	3.31E-01	5.13E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	8.33E-04	5.56E-05	4.10E-06	3.11E-05	5.70E-05	6.96E-05	8.14E-05
		7.39E-06	1.88E-06	5.26E-06	8.24E-06	1.15E-05	2.51E-05
HY	8.33E-04	8.35E-05	(9.40E-06)				
RCV-RV	5.00E-01	2.33E-01	1.74E-02	1.47E-01	3.01E-01	3.92E-01	4.81E-01
		2.91E-02	8.02E-03	2.40E-02	3.98E-02	5.54E-02	1.16E-01
HY-RV	5.00E-01	3.50E-01	3.55E-01	3.94E-01	4.42E-01	4.71E-01	5.10E-01
		3.66E-02	3.71E-02	4.11E-02	4.62E-02	5.23E-02	1.07E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.11E-01	6.67E-01	9.04E-01
		2.11E-01	1.18E-01	2.03E-01	3.02E-01	4.34E-01	1.21E+00
SIML-SIML	5.00E-01	5.09E-01	5.01E-01	5.10E-01	5.12E-01	5.28E-01	5.23E-01
		2.40E-01	1.24E-01	2.04E-01	2.72E-01	3.35E-01	5.04E-01

3-4 : Estimation of hedging coefficient: Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	8.33E-04	7.16E-05	3.55E-05	8.70E-05	8.57E-05	8.71E-05	8.51E-05
		5.44E-04	3.83E-04	2.09E-04	1.25E-04	9.14E-05	5.88E-05
HY	8.33E-04	8.28E-05	(3.86E-04)				
RCV-RV	5.00E-01	9.90E-04	7.81E-04	1.18E-02	3.33E-02	6.35E-02	2.09E-01
		7.53E-03	8.39E-03	2.84E-02	4.89E-02	6.64E-02	1.39E-01
HY-RV	5.00E-01	1.15E-03	1.81E-03	1.13E-02	3.18E-02	6.04E-02	2.09E-01
		5.35E-03	8.45E-03	5.26E-02	1.51E-01	2.85E-01	1.00E+00
HY-SIML	5.00E-01	5.09E-01	5.04E-01	5.13E-01	5.20E-01	5.60E-01	9.17E-01
		2.39E+00	2.37E+00	2.69E+00	3.02E+00	3.47E+00	6.06E+00
SIML-SIML	5.00E-01	4.89E-01	4.83E-01	4.96E-01	5.08E-01	5.15E-01	5.18E-01
		1.47E-01	1.32E-01	2.05E-01	2.72E-01	3.21E-01	5.20E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	6.27E-02	4.93E-02	3.15E-01	4.23E-01	4.58E-01	4.94E-01
		7.32E-03	8.08E-03	2.29E-02	3.72E-02	5.20E-02	1.19E-01
HY-RV	5.00E-01	9.39E-02	1.34E-01	3.50E-01	4.38E-01	4.68E-01	5.10E-01
		6.65E-03	9.40E-03	2.56E-02	3.80E-02	4.84E-02	1.02E-01
HY-SIML	5.00E-01	5.23E-01	5.23E-01	5.58E-01	5.94E-01	6.40E-01	8.17E-01
		1.14E-01	1.14E-01	2.00E-01	3.00E-01	4.04E-01	9.37E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.28E-01	5.32E-01	5.40E-01
		1.42E-01	1.26E-01	2.01E-01	2.65E-01	3.15E-01	4.87E-01

3-5 : Estimation of hedging coefficient: Case 3 (Stochastic volatility; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		7.23E-05	4.27E-06	3.79E-05	6.44E-05	8.49E-05	9.93E-05
		1.74E-04	5.33E-05	1.24E-04	1.20E-04	9.15E-05	6.27E-05
HY		9.83E-05	(1.22E-04)				
RCV-RV	5.00E-01	9.78E-03	6.24E-04	7.99E-03	2.61E-02	6.18E-02	2.28E-01
		2.35E-02	7.57E-03	2.61E-02	4.85E-02	6.64E-02	1.40E-01
HY-RV	5.00E-01	1.33E-02	1.40E-02	2.07E-02	3.99E-02	7.16E-02	2.31E-01
		1.65E-02	1.73E-02	2.58E-02	4.97E-02	8.97E-02	3.01E-01
HY-SIML	5.00E-01	5.34E-01	4.58E-01	5.23E-01	5.86E-01	6.25E-01	8.47E-01
		7.17E-01	5.96E-01	6.97E-01	8.44E-01	9.46E-01	1.76E+00
SIML-SIML	5.00E-01	5.03E-01	4.53E-01	5.01E-01	5.11E-01	5.08E-01	5.02E-01
		2.41E-01	1.30E-01	2.06E-01	2.85E-01	3.46E-01	5.11E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		6.69E-05	4.79E-06	3.67E-05	6.83E-05	8.35E-05	9.73E-05
		8.82E-06	2.22E-06	6.09E-06	9.65E-06	1.32E-05	2.83E-05
HY		1.00E-04	(1.13E-05)				
RCV-RV	5.00E-01	2.46E-01	1.78E-02	1.50E-01	3.07E-01	3.96E-01	4.81E-01
		3.06E-02	8.24E-03	2.42E-02	3.99E-02	5.39E-02	1.13E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.08E-01	4.51E-01	4.76E-01	5.09E-01
		3.76E-02	3.81E-02	4.15E-02	4.58E-02	5.26E-02	1.02E-01
HY-SIML	5.00E-01	5.68E-01	5.20E-01	5.64E-01	6.18E-01	6.50E-01	8.19E-01
		2.17E-01	1.15E-01	2.09E-01	3.25E-01	3.83E-01	8.85E-01
SIML-SIML	5.00E-01	5.15E-01	4.99E-01	5.17E-01	5.21E-01	5.17E-01	5.16E-01
		2.39E-01	1.23E-01	2.01E-01	2.74E-01	3.34E-01	5.08E-01

3-6 : Estimation of coefficient coefficient: Case 3 (Stochastic volatility; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		8.23E-05	4.50E-05	7.87E-05	8.71E-05	9.35E-05	9.86E-05
		5.36E-04	3.80E-04	2.10E-04	1.25E-04	9.46E-05	6.15E-05
HY		1.31E-04	(3.86E-04)				
RCV-RV	5.00E-01	1.14E-03	9.88E-04	1.07E-02	3.37E-02	6.71E-02	2.25E-01
		7.42E-03	8.31E-03	2.84E-02	4.86E-02	6.79E-02	1.37E-01
HY-RV	5.00E-01	1.81E-03	2.86E-03	1.76E-02	5.05E-02	9.24E-02	3.11E-01
		5.34E-03	8.43E-03	5.20E-02	1.49E-01	2.78E-01	9.22E-01
HY-SIML	5.00E-01	6.79E-01	6.76E-01	7.17E-01	7.83E-01	8.27E-01	1.04E+00
		2.01E+00	2.01E+00	2.18E+00	2.51E+00	2.83E+00	4.02E+00
SIML-SIML	5.00E-01	4.93E-01	4.89E-01	4.98E-01	4.98E-01	4.95E-01	4.78E-01
		1.40E-01	1.26E-01	1.97E-01	2.68E-01	3.21E-01	4.85E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		6.68E-05	3.70E-05	8.99E-05	9.64E-05	9.84E-05	1.01E-04
		7.08E-06	5.24E-06	6.89E-06	9.77E-06	1.34E-05	2.86E-05
HY		1.00E-04	(6.32E-06)				
RCV-RV	5.00E-01	7.26E-02	5.65E-02	3.31E-01	4.31E-01	4.65E-01	4.94E-01
		7.74E-03	8.00E-03	2.29E-02	3.65E-02	5.14E-02	1.11E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.70E-01	4.50E-01	4.77E-01	5.09E-01
		6.89E-03	9.63E-03	2.45E-02	3.55E-02	4.75E-02	9.81E-02
HY-SIML	5.00E-01	5.21E-01	5.21E-01	5.51E-01	5.92E-01	6.28E-01	8.22E-01
		1.11E-01	1.10E-01	1.90E-01	2.75E-01	3.66E-01	7.92E-01
SIML-SIML	5.00E-01	5.05E-01	5.03E-01	5.12E-01	5.12E-01	5.11E-01	5.04E-01
		1.38E-01	1.22E-01	1.92E-01	2.64E-01	3.19E-01	4.99E-01

Table 5.1 : Estimation of hedging coefficient: Case 4 (ACD; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.82E-05	3.41E-06	3.80E-05	7.20E-05	8.57E-05	9.75E-05
		1.72E-04	5.30E-05	1.25E-04	1.24E-04	9.39E-05	6.17E-05
HY	1.00E-04	1.05E-04					
		1.23E-04					
RCV-RV	5.00E-01	9.13E-03	4.78E-04	8.08E-03	2.94E-02	6.17E-02	2.24E-01
		2.32E-02	7.47E-03	2.64E-02	5.03E-02	6.74E-02	1.38E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.24E-02	4.32E-02	7.68E-02	2.51E-01
		1.66E-02	1.75E-02	2.62E-02	5.06E-02	8.99E-02	3.00E-01
HY-SIML	5.00E-01	5.90E-01	4.92E-01	5.78E-01	6.48E-01	6.87E-01	8.83E-01
		7.53E-01	5.98E-01	7.33E-01	9.13E-01	1.05E+00	1.74E+00
SIML-SIML	5.00E-01	5.07E-01	4.50E-01	4.96E-01	5.06E-01	5.16E-01	4.94E-01
		2.46E-01	1.28E-01	2.08E-01	2.73E-01	3.23E-01	4.92E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.86E-05	4.69E-06	3.60E-05	6.72E-05	8.27E-05	9.80E-05
		9.08E-06	2.15E-06	6.40E-06	9.82E-06	1.30E-05	2.82E-05
HY	1.00E-04	1.01E-04					
		1.12E-05					
RCV-RV	5.00E-01	2.51E-01	1.74E-02	1.47E-01	3.02E-01	3.91E-01	4.83E-01
		3.18E-02	7.98E-03	2.51E-02	4.04E-02	5.32E-02	1.11E-01
HY-RV	5.00E-01	3.68E-01	3.74E-01	4.10E-01	4.52E-01	4.77E-01	5.10E-01
		3.82E-02	3.86E-02	4.14E-02	4.69E-02	5.47E-02	1.04E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.14E-01	6.59E-01	8.82E-01
		2.09E-01	1.17E-01	2.04E-01	3.10E-01	4.18E-01	1.07E+00
SIML-SIML	5.00E-01	5.19E-01	5.02E-01	5.16E-01	5.24E-01	5.34E-01	5.19E-01
		2.45E-01	1.21E-01	2.05E-01	2.77E-01	3.25E-01	5.13E-01

Table 5.2 : Estimation of hedging coefficient: Case 4 (ACD ; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	7.51E-05	3.09E-05	8.39E-05	9.73E-05	1.04E-04	1.00E-04
		5.01E-04	3.78E-04	2.16E-04	1.26E-04	9.30E-05	6.06E-05
HY	1.00E-04	1.31E-04					
		3.80E-04					
RCV-RV	5.00E-01	1.04E-03	6.88E-04	1.13E-02	3.75E-02	7.38E-02	2.32E-01
		6.94E-03	8.36E-03	2.91E-02	4.86E-02	6.60E-02	1.39E-01
HY-RV	5.00E-01	1.81E-03	2.90E-03	1.78E-02	5.08E-02	9.60E-02	3.10E-01
		5.26E-03	8.41E-03	5.14E-02	1.48E-01	2.72E-01	9.22E-01
HY-SIML	5.00E-01	6.77E-01	6.76E-01	7.16E-01	8.01E-01	8.07E-01	1.03E+00
		2.00E+00	1.98E+00	2.14E+00	2.47E+00	2.68E+00	4.58E+00
SIML-SIML	5.00E-01	4.90E-01	4.85E-01	4.98E-01	5.09E-01	5.21E-01	5.24E-01
		1.46E-01	1.28E-01	2.06E-01	2.75E-01	3.46E-01	5.14E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.84E-05	3.59E-05	8.92E-05	9.69E-05	9.93E-05	1.01E-04
		6.91E-06	5.18E-06	6.92E-06	1.02E-05	1.34E-05	2.88E-05
HY	1.00E-04	1.00E-04					
		6.34E-06					
RCV-RV	5.00E-01	7.44E-02	5.53E-02	3.28E-01	4.31E-01	4.67E-01	5.01E-01
		7.53E-03	7.97E-03	2.32E-02	3.73E-02	5.14E-02	1.14E-01
HY-RV	5.00E-01	1.09E-01	1.55E-01	3.70E-01	4.49E-01	4.76E-01	5.16E-01
		7.11E-03	9.88E-03	2.53E-02	3.80E-02	4.97E-02	1.02E-01
HY-SIML	5.00E-01	5.25E-01	5.26E-01	5.60E-01	6.15E-01	6.58E-01	8.42E-01
		1.16E-01	1.16E-01	1.93E-01	3.17E-01	4.24E-01	8.77E-01
SIML-SIML	5.00E-01	5.03E-01	5.02E-01	5.13E-01	5.25E-01	5.30E-01	5.35E-01
		1.42E-01	1.25E-01	1.99E-01	2.68E-01	3.29E-01	4.94E-01

Table 5.3 : Estimation of hedging coefficient: Case 5 $(g = 0.2; \lambda = 1800)$

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	9.97E-06	2.88E-07	7.30E-06	2.12E-05	4.27E-05	8.36E-05
		1.03E-04	2.96E-05	8.65E-05	1.34E-04	1.55E-04	1.21E-04
HY	1.00E-04	1.74E-05					
		1.16E-04					
RCV-RV	5.00E-01	2.46E-03	9.30E-05	1.98E-03	6.85E-03	1.76E-02	1.01E-01
		2.57E-02	7.48E-03	2.37E-02	4.37E-02	6.41E-02	1.48E-01
HY-RV	5.00E-01	4.32E-03	4.35E-03	4.74E-03	5.56E-03	6.97E-03	2.12E-02
		2.89E-02	2.91E-02	3.17E-02	3.78E-02	4.83E-02	1.47E-01
HY-SIML	5.00E-01	7.18E-02	4.00E-02	6.80E-02	7.64E-02	1.08E-01	1.45E-01
		6.02E-01	3.01E-01	5.87E-01	7.59E-01	9.28E-01	1.54E+00
SIML-SIML	5.00E-01	4.25E-01	2.26E-01	4.11E-01	4.72E-01	4.98E-01	4.92E-01
		2.45E-01	1.54E-01	2.23E-01	2.83E-01	3.34E-01	5.17E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	1.34E-05	9.91E-07	9.43E-06	2.49E-05	4.19E-05	8.38E-05
		2.48E-06	4.92E-07	1.99E-06	4.52E-06	7.85E-06	2.59E-05
HY	1.00E-04	2.01E-05					
		3.54E-06					
RCV-RV	5.00E-01	2.15E-01	1.56E-02	1.26E-01	2.58E-01	3.51E-01	4.75E-01
		3.73E-02	7.73E-03	2.48E-02	4.16E-02	5.69E-02	1.16E-01
HY-RV	5.00E-01	3.22E-01	3.16E-01	2.67E-01	2.09E-01	1.68E-01	1.17E-01
		5.28E-02	5.14E-02	4.27E-02	3.28E-02	2.63E-02	2.44E-02
HY-SIML	5.00E-01	1.14E-01	1.09E-01	1.13E-01	1.22E-01	1.33E-01	1.80E-01
		4.21E-02	2.48E-02	4.01E-02	5.97E-02	8.67E-02	2.41E-01
SIML-SIML	5.00E-01	5.10E-01	4.93E-01	5.09E-01	5.13E-01	5.29E-01	5.23E-01
		2.39E-01	1.24E-01	2.04E-01	2.71E-01	3.34E-01	4.97E-01

6 Conclusion

1. The SIML estimator is simple and it has reasonable statistical properties.
2. We have the asymptotic robustness in the sense that it is consistent and it has the asymptotic normality (in a proper sense) under a fairly general conditions. They include not only the cases when the micro-market noises are possibly autocorrelated, we have non-linear price adjustments including the round-off errors and the high-frequency data are randomly sampled.
3. The SIML estimator is also simple and useful for multivariate high frequency series including the estimation of integrated covariances and the hedging coefficient.

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