THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

Rank tests for short memory stationarity

Pranab K. Sen
jointly with Matteo M. Pelagatti

University of North Carolina at Chapel Hill
Università degli Studi di Milano-Bicocca

50th Anniversary of the Department of Statistics
University of Connecticut, Storrs

Outline

Motivation

One of us was working with time series of electricity prices and found that:

- in many paper prices were found (or held as) stationary and this is quite strange as they depend on gas and oil prices which are usually well approximated by integrated processes (in logs);
- due to technical reasons electricity prices are extremely volatile and so the nonstationary signal is buried into a volatile and leptokurtic noise;
- most unit-root and the KPSS stationarity tests are optimal under Gaussianity and fail to find nonstationarity when data are leptokurtic and second moments may not exist.

The Index KPSS test

An article inspired our idea for robust stationarity tests

- de Jong *et al.* (2007, J.Econometrics) prove that the KPSS test applied to the **sign of the median-centered observations** (IKPSS) has the same asymptotic distribution under the null as the standard KPSS.
- IKPSS PRO: existence of moments not required, good power under extremely fat-tailed distribution.
- IKPSS CON: under Gaussianity or moderate excess kurtosis significant loss of power when compared to KPSS.
- de Jong *et al.* (2007) do not provide a test for trend-stationarity (stationarity on a linear trend), whereas time series analysts are usually interested in this hypothesis.

The KPSS test in two slides

Suppose that for $t = 1, \dots, T$

$$X_t = \mu_t + \varepsilon_t$$
$$\mu_t = \mu_{t-1} + \zeta_t$$

with ε_t and ζ_t i.i.d. zero-mean processes with variances $\sigma_\varepsilon^2 > 0$ and $\sigma_\zeta^2 \geq 0$. Under Gaussianity, the locally best invariant (LBI) test for the hypothesis $\sigma_\zeta^2 = 0$ is (Nabeya, Tanaka 1988 Annals of Statistics)

$$\mathrm{LBI}_T := \frac{1}{\hat{\sigma}_\varepsilon^2} \sum_{t=1}^{T} S_t^2$$

where

$$e_t := X_t - \bar{X}_T, \quad S_t := \sum_{s=1}^{t} e_s, \quad \hat{\sigma}_\varepsilon^2 := \frac{1}{T} S_T^2.$$

Under the null $\mathrm{LBI}_T / T^2 \Rightarrow \int V(r)^2 \, \mathrm{d}r$, where V is a standard Brownian bridge on $[0, 1]$.

Kwiatowski, Phillips, Schmidt & Shin (KPSS) show that if we relax the assumption of normality of ε_t and ζ_t to the existence of second moments and the i.i.d.-ness of ε_t to (strong) mixing stationarity and the existence of the *long-run variance*

$$\sigma^2 := \lim_{t\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{t=1}^{T} \varepsilon_t\right]^2$$

then

$$\eta_T := \frac{1}{T\hat{\sigma}_T^2}\sum_{t=1}^{T} S_t^2 \Rightarrow \int_0^1 V(r)^2\,\mathrm{d}r,$$

where $\hat{\sigma}_T^2$ is the consistent estimator of the long-run variance

$$\hat{\sigma}_T^2 := \frac{1}{T}\sum_{s=1}^{T}\sum_{t=1}^{T} k\left(\frac{s-t}{\gamma_T}\right) e_s e_t,$$

with k kernel function with bandwidth γ_T such that $\gamma_T \to \infty$ as T diverges and $\gamma_T = \mathrm{o}(\sqrt{T})$.

The Rank KPSS test

Let the observed time series be a sample path of the real random sequence $\{X_1, \ldots, X_T\}$ and let

$$R_{T,t} = \sum_{i=1}^{T} \mathbb{I}_{\{X_i \leq X_t\}}, \qquad \text{for } t = 1, \ldots, T, \tag{1}$$

with $\mathbb{I}_A$ indicator function of the set A, be the *rank* of X_t among $\{X_1, \ldots, X_T\}$.

Notice that the arithmetic mean of the rank sequence $\{R_{T,1}, \ldots, R_{T,T}\}$ is $(T+1)/2$ and does not depend on the data.

The test statistic we propose in this paper is the KPSS applied to the ranks of the observations.

Partial sums of ranks

So, let $S_{T,t}$ be the sequence of demeaned partial sums:

$$S_{T,t} = \sum_{i=1}^{t} \left(\frac{R_{T,i}}{T} - \frac{T+1}{2T} \right). \qquad (2)$$

Notice that the KPSS statistic is invariant to scale transformations, so working with $R_{T,i}/T$ rather than $R_{T,i}$ turns out to generate the same statistic. We chose to work on the former form since under stationarity this makes our partial sum process diverge at the same rate as the analogous quantity defined in Kwiatowski et al. (1992).

The RKPSS statistic

In complete analogy with Kwiatowski et al. (1992), define

$$\eta_T^R = T^{-2} \sum_{i=t}^{T} S_{T,t}^2 \tag{3}$$

and the rank KPSS (RKPSS) test statistic as

$$\hat{\eta}_T^R = \frac{\eta_T^R}{\hat{\sigma}_T^2}, \tag{4}$$

where $\hat{\sigma}_T^2$ is a kernel estimator of the long-run variance of $\{R_{T,t}/T\}$:

$$\hat{\sigma}_T^2 = \frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} k\left(\frac{s-t}{\gamma_T}\right) \left[\frac{R_{T,s}}{T} - \frac{T+1}{2T}\right] \left[\frac{R_{T,t}}{T} - \frac{T+1}{2T}\right], \tag{5}$$

with $k(\cdot)$ symmetric kernel function and γ_T bandwidth parameter.

Null Hypothesis & Kernel function

Assumption 1. (Short memory stationarity)

1. $\{X_1, \ldots, X_T\}$ is a strictly stationary random sequence.
2. $\{X_1, \ldots, X_T\}$ is strong mixing with $\alpha(T) = O(T^{-v})$, $v > 2$.
3. For all $i \in \{1, \ldots, T\}$ and $T \in \mathbb{N}$, X_i has non-degenerate absolutely continuous distribution function $F(\cdot)$ defined on $\mathbb{R}$ with density $f(\cdot)$.

Assumption 2. (Regularity of the kernel function)

1. $k(\cdot)$ satisfies $\int_{-\infty}^{\infty} |\psi(z)| \, \mathrm{d}z < \infty$, $\psi(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} k(x) \exp(-\mathrm{i}zx) \, \mathrm{d}x$.
2. $k(\cdot)$ is continuous at all but a finite number of points, $k(x) = k(-x)$, $|k(x)| < l(x)$ where $l(x)$ is non-increasing and $\int_0^{\infty} |l(x)| \, \mathrm{d}x < \infty$, and $k(0) = 1$.
3. $\gamma_T / \sqrt{T} \to 0$ and $\gamma_T \to \infty$ as $T \to \infty$.

Spearman's rank autocorrelation coefficient

$$\rho_{i,j} = 12\mathbb{E}\left\{ \left[F(X_{T,i}) - 1/2\right] \left[F(X_{T,j}) - 1/2\right] \right\}$$

Asymptotics under the Null

Theorem (Distribution under short-memory stationarity)

Under Assumption 1,

$$\eta^R_{\mu,T} \Rightarrow \sigma^2 \int_0^1 V(r)^2 \, \mathrm{d}r,$$

with V standard Brownian bridge and $\sigma^2 = \frac{1}{12}\left[1 + 2\sum_{k=2}^{\infty} \rho_{1,k}\right]$; furthermore

$$T^{-1/2} S_{T,t} = T^{-1/2} \left\{ \sum_{i=1}^{t} F(X_i) - \frac{t}{T} \sum_{i=1}^{T} F(X_i) \right\} + \mathrm{O}_p(T^{-1/2}).$$

Under Assumptions 1 and 2,

$$\hat{\eta}^R_{\mu,T} \Rightarrow \int_0^1 V(r)^2 \, \mathrm{d}r.$$

Asymptotics under the alternative of integration

Theorem (Distrib. under integration possibly after monotone transform)

Suppose there exists a strictly monotone (Borel) function $g : \mathbb{R} \mapsto \mathbb{R}$ such that $T^{-1/2} g(X_{\lfloor rT \rfloor, T}) \Rightarrow \omega W(r)$, where ω is a strictly positive real number and W is standard Brownian motion on $[0, 1]$, then

$$\frac{\eta^R_{\mu,T}}{T} \Rightarrow \int_0^1 \left[\int_0^s R_0(r)\,\mathrm{d}r \right]^2 \mathrm{d}s,$$

with $R_0(r) = \int_0^1 \mathbb{I}_{\{W(u)<W(r)\}}\,\mathrm{d}u - \frac{1}{2}$, and

$$\hat{\sigma}^2_T \leq \frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} k\left(\frac{s-t}{\gamma_T}\right) = O(\gamma_T).$$

Remarks

Corollary

The RKPSS statistic $\hat{\eta}^R_{\mu,T}$ is consistent against I(1)-ness.

- The alternative hypothesis we used is much weaker than the corresponding hypothesis for the KPSS statistic. While for the KPSS test, the process $X_{T,t}$ must be I(1), in the RKPSS case the I(1) process can be any strictly monotonic transformation of $X_{T,t}$.
- Theorem 2 suggests that the statistic η^R_T/T can be used to test the hypothesis $g(X_{T,t}) \sim \mathrm{I}(1)$ against stationarity.
  Indeed, η^R_T/T is asymptotically free of nuisance parameters and converges weakly to a proper distribution under the null and to the Dirac (point mass) measure concentrated at zero under the alternative.

Asymptotic relative efficiency

Consider the local alternative

$$Y_t = \frac{\sigma_z}{T} \underbrace{\sum_{s=1}^{t} Z_t}_{\text{integrated}} + \underbrace{X_t}_{\text{stationary}}, \qquad t = 1, 2, \ldots, T,$$

where σ_z and σ_x are positive real numbers, and Z_t and X_t are mutually independent stationary processes such that, for $r \in [0, 1]$,

$$T^{-1/2} \sum_{i=1}^{\lfloor rT \rfloor} Z_t \Rightarrow W_z(r) \quad \text{and} \quad T^{-1/2} \sum_{i=1}^{\lfloor rT \rfloor} X_t \Rightarrow \sigma_x W_x(r).$$

with W_z and W_x independent standard Brownian motions.

Asymptotic relative efficiency (cont.)

Define the partial sum processes of the KPSS and RKPSS statistic as

$$S^K_{T,t} := \sum_{s=1}^{t} (Y_s - \bar{Y}_T),$$

$$S^R_{T,t} := \sum_{s=1}^{t} \left(\frac{R^y_{T,t}}{T} - \frac{T+1}{2T}. \right)$$

Asymptotics under local alternatives

Theorem (A.R.E. of the RKPSS with respect to the KPSS)

Assume that Y_t is generated by the above local alternative, where X_t satisfies Assumption 1, then, for $r \in [0, 1]$,

$$\frac{1}{\sqrt{T}\sigma_x} S^K_{T,\lfloor rT \rfloor} \Rightarrow V(r) + \frac{\sigma_z}{\sigma_x} K(r)$$
$$\frac{1}{\sqrt{T}\sigma} S^R_{T,\lfloor rT \rfloor} \Rightarrow V(r) + f_2(0)\frac{\sigma_z}{\sigma} K(r)$$

where $f_2(0) := \mathbb{E}\, f(X)$ and $V(r)$ is a standard Brownian bridge independent of $K(r) := \int_0^r W_z(u)\,\mathrm{d}u - r\int_0^1 W_z(u)\,\mathrm{d}u$.
The asymptotic relative efficiency of the RKPSS test with respect to the KPSS is

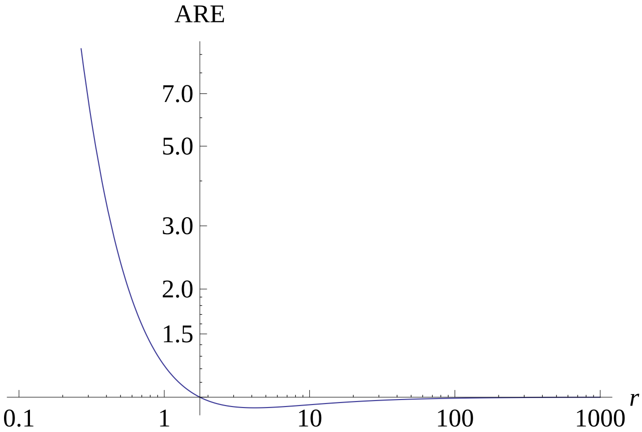
$$e_{R,K} = f_2(0)\frac{\sigma_x}{\sigma}.$$

ARE under serial independence of X_t and Z_t

Distribution	$f_2(0)$	$e_{\mathrm{R,K}}$
Normal	$\frac{1}{2\sqrt{\pi}}$	0.977
Uniform	$\frac{1}{\sqrt{12}}$	1.000
Logistic	$\frac{\pi}{6\sqrt{3}}$	1.047
Student5	$\frac{7}{4\sqrt{3}\pi}$	1.114
Laplace	$\frac{1}{2\sqrt{2}}$	1.225
Student3	$\frac{5}{4\pi}$	1.378

For Student's t, the ARE diverges as the df approach 2.

ARE for the Generalized Error or Exponential Power Distr.



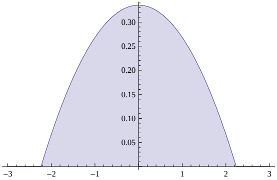
r is the shape parameter ($r = 2$ Gauss, $r = 1$ Laplace, $r = \infty$ Uniform)

The function reaches its minimum at point $(4.193, 0.934)$.

The bounds of the ARE?

- The ARE is unbounded from above.
- The ARE is bounded from below by 0.930.
- The least favorable density was found by Hodges and Lehman (1956):

$$f(x) = \begin{cases} \frac{3}{20\sqrt{5}}(5 - x^2), & \text{for } x^2 \leq 5 \\ 0, & \text{for } x^2 > 5 \end{cases}$$

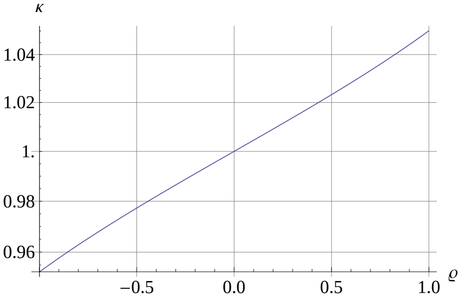


ARE under dependence

- Under dependence, the ARE values must be multiplied by

$$\kappa := \sqrt{\left(1 + 2\sum_{i=1}^{\infty} \rho_{1,i}\right) \Big/ \left(1 + 2\sum_{i=1}^{\infty} \varrho_{1,i}\right)},$$

  with $\rho_{i,j}$ Spearman's and $\varrho_{i,j}$ Pearson's correlation.
- If X_t is Gaussian AR(1) then



KPSS for (linear) trend stationarity

Suppose the data generating process is

$$Y_t = \alpha + \beta t + X_t,$$

with X_t short memory stationary sequence.

- The KPSS applied to least-squares detrended data converges to $\int V_2(r)^2 \, \mathrm{d}r$ under the null, with V_2 second-level (or detrended) Brownian bridge.
- Is there a robust rank-based way to detrend data so that our RKPSS converges to $\int V_2(r)^2 \, \mathrm{d}r$ as well?
- R-estimates or the asymptotically equivalent Theil-Sen regression estimator:

$$\tilde{\beta}_T := \mathrm{median}\left\{ \frac{Y_j - Y_i}{j - i};\ 1 \leq i < j \leq T \right\}.$$

Theil-Sen estimator (TSE) asymptotics under dependence

Theorem (Asymptotic distribution of the TSE)

Let the linear model hold with X_t strictly stationary and ergodic having a continuous distribution with density f. Then, $\tilde{\beta}_T$ is consistent for β. Furthermore, if the regression errors $\{X_t\}$ are strong mixing with mixing coefficients $\sum_{n=1}^{\infty} \alpha(n) < \infty$, and $f_2(0) := \int f(x)^2 \, \mathrm{d}x < \infty$, then

$$Q_T(\tilde{\beta}_T - \beta) \Rightarrow \mathrm{N}\left(0, \frac{\sigma^2}{f_2(0)^2}\right),$$

where $Q_T := \sqrt{T(T^2-1)/12}$.
Finally, under the same conditions and $\bar{t} = (T+1)/2$,

$$Q_T(\tilde{\beta}_T - \beta) = \frac{\sqrt{12}}{T^{3/2} f_2(0)} \sum_{t=1}^{T} [F(X_t) - 1/2]\,(t - \bar{t}) + \mathrm{o}_p(1).$$

Rank KPSS test after Theil-Sen detrending

Theorem (Distribution under trend-stationarity)

Let $\eta^R_{\tau,T}$ and $\hat{\eta}^R_{\tau,T}$ the RKPSS statistics applied to $Y_t - \tilde{\beta}_T t$. Under the linear model, Assumption 1 for the regression errors and $f_2(0) := \int f(x)^2 \,\mathrm{d}x < \infty$,

$$\eta^R_{\tau,T} \Rightarrow \sigma \int V_2(r)^2 \,\mathrm{d}r,$$

where $V_2(r)$ is a second-level Brownian bridge.
Under the above assumptions and Assumptions 2,

$$\hat{\eta}^R_{\tau,T} \Rightarrow \int V_2(r)^2 \,\mathrm{d}r.$$

Rank KPSS test after Theil-Sen detrending

Theorem (Distribution of TSE and RKPSS under integration)

Let $Y_t = \beta t + X_t$ *with* $T^{-1/2} X_{\lfloor rT \rfloor} \Rightarrow \omega W(r)$, $r \in [0,1]$, *where* ω *is a positive real number and* W *a standard Brownian motion on* $[0,1]$, *then*

i) $\tilde{\beta}_T \xrightarrow{\mathrm{p}} \beta$;

ii) $T^{1/2}(\tilde{\beta}_T - \beta) \Rightarrow H$, *where* H *is a random variable with an absolutely continuous distribution symmetric about zero;*

iii)

$$\frac{\eta^R_{\tau,T}}{T} \Rightarrow \int_0^1 \left[\int_0^s \tilde{R}_0(r)\,\mathrm{d}r \right]^2 \mathrm{d}s,$$

with $\tilde{R}_0(r) = \int_0^1 \mathbb{I}_{\{W(u) - Hu \leq W(r) - Hr\}}\,\mathrm{d}u - \frac{1}{2}$ *and* $r \in [0,1]$;

iv)

$$\hat{\sigma}^2_T \leq \frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} k\left(\frac{s-t}{\gamma_T}\right) = O(\gamma_T).$$

Monte-Carlo experiments design

We reproduce the design of de Jong et al. (2007).

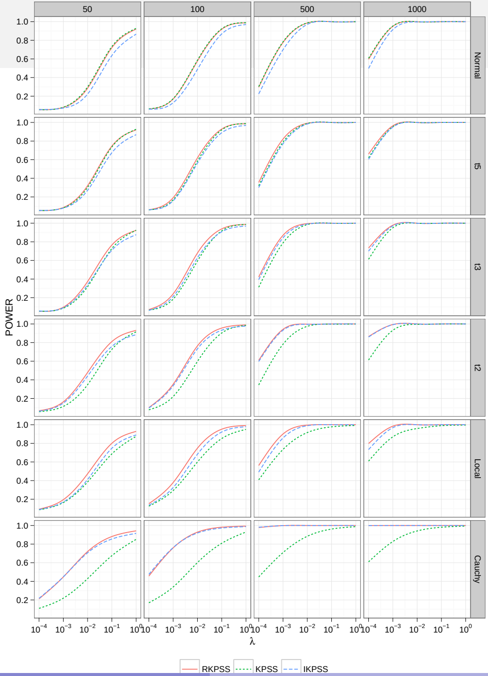
- 20,000 replications.
- Sample sizes ranging from $T = 50$ to $T = 5000$.
- Distributions: Normal, Student's t_5, t_3, t_2, t_1 (Cauchy) and *local to finite variance* $x_t = x_{1,t} + T^{-1/2} x_{2,t}$, where x_1 is normal and x_2 Cauchy.
- Under I(0): white noise and AR(1) processes with $\phi = 0.5$.
- Under I(1): $\mu_0 = 0, \quad \mu_t = \mu_{t-1} + \sqrt{\lambda}\,\eta_t, \quad x_t = \mu_t + \varepsilon_t$, where λ is the signal-to-noise ratio and ranges between 0.0001 and 1.
- Kernel: Bartlett with $\gamma_T = 4(T/100)^{1/4}$ (white noise case), $\gamma_T = 1.1447 \cdot (1.7778 \cdot T)^{1/3}$ (AR(1) case, Andrews 1991).

Size: (a) no Kernel, (b) with Kernel.

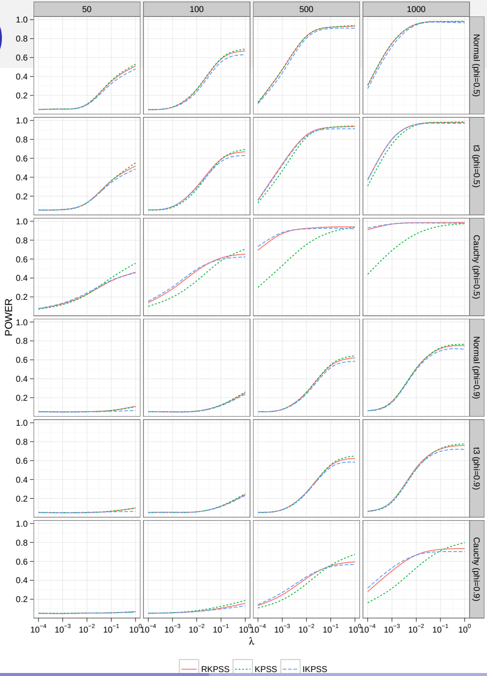
(a)	Normal			t_5			t_3			t_2			Local			Cauchy		
T	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS
50	0.052	0.052	0.053	0.052	0.055	0.052	0.048	0.053	0.052	0.044	0.052	0.052	0.037	0.052	0.049	0.026	0.052	0.054
100	0.049	0.049	0.051	0.048	0.049	0.051	0.049	0.051	0.050	0.044	0.051	0.053	0.039	0.050	0.049	0.030	0.051	0.051
200	0.052	0.052	0.052	0.048	0.049	0.049	0.050	0.049	0.049	0.042	0.048	0.050	0.037	0.051	0.051	0.027	0.050	0.049
500	0.052	0.050	0.050	0.052	0.051	0.049	0.049	0.050	0.051	0.046	0.050	0.052	0.038	0.049	0.049	0.028	0.051	0.051
1000	0.051	0.050	0.052	0.051	0.050	0.048	0.051	0.049	0.051	0.046	0.050	0.050	0.038	0.049	0.051	0.030	0.053	0.052
2000	0.050	0.050	0.050	0.049	0.049	0.048	0.049	0.049	0.049	0.047	0.052	0.052	0.040	0.052	0.052	0.028	0.050	0.048
5000	0.053	0.051	0.053	0.050	0.049	0.050	0.050	0.049	0.048	0.047	0.048	0.050	0.039	0.049	0.050	0.027	0.047	0.050

(b)	Normal, $\rho = .5$			$t_3, \rho = .5$			Cauchy, $\rho = .5$			Normal, $\rho = .9$			$t_3, \rho = .9$			Cauchy, $\rho = .9$		
T	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS
50	0.063	0.066	0.063	0.062	0.069	0.064	0.045	0.086	0.082	0.027	0.030	0.024	0.028	0.028	0.023	0.025	0.033	0.021
100	0.069	0.070	0.066	0.067	0.072	0.068	0.048	0.087	0.084	0.028	0.031	0.033	0.027	0.031	0.035	0.023	0.034	0.035
200	0.069	0.070	0.067	0.069	0.074	0.071	0.044	0.077	0.072	0.068	0.072	0.073	0.066	0.072	0.073	0.048	0.092	0.095
500	0.066	0.066	0.064	0.064	0.065	0.064	0.039	0.072	0.070	0.077	0.080	0.078	0.075	0.079	0.076	0.054	0.096	0.092
1000	0.064	0.063	0.060	0.061	0.062	0.062	0.038	0.067	0.065	0.073	0.073	0.071	0.071	0.072	0.072	0.051	0.087	0.085
2000	0.059	0.060	0.059	0.061	0.062	0.059	0.037	0.064	0.064	0.070	0.070	0.068	0.070	0.072	0.069	0.049	0.081	0.077
5000	0.060	0.059	0.058	0.057	0.057	0.058	0.032	0.060	0.058	0.064	0.064	0.063	0.066	0.065	0.064	0.043	0.074	0.072

Size-adjusted power IID



Size-adjusted power AR(1)

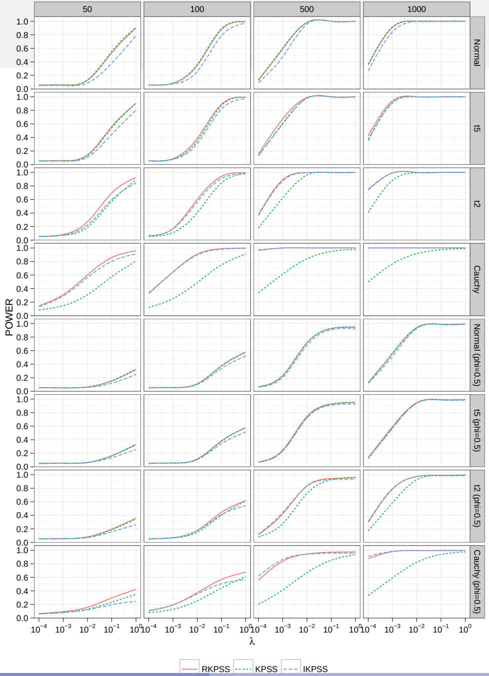


Size for detrended tests

T	Normal			t_5			t_2			Cauchy		
	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS
(a) i.i.d. data												
50	0.053	0.053	0.096	0.051	0.054	0.098	0.041	0.054	0.100	0.026	0.053	0.097
100	0.049	0.051	0.066	0.049	0.051	0.066	0.041	0.048	0.067	0.026	0.047	0.065
200	0.051	0.051	0.059	0.050	0.050	0.056	0.047	0.053	0.059	0.026	0.050	0.056
500	0.047	0.048	0.051	0.048	0.049	0.055	0.046	0.049	0.051	0.026	0.051	0.052
1000	0.051	0.050	0.051	0.052	0.052	0.052	0.045	0.047	0.051	0.026	0.050	0.052

T	Normal			t_5			t_2			Cauchy		
	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS	KPSS	RKPSS	IKPSS
(b) AR(1) with $\phi = 0.5$												
50	0.080	0.085	0.126	0.074	0.093	0.128	0.081	0.088	0.125	0.071	0.129	0.154
100	0.080	0.082	0.088	0.078	0.092	0.100	0.081	0.086	0.091	0.066	0.114	0.111
200	0.080	0.080	0.079	0.072	0.083	0.082	0.079	0.078	0.081	0.060	0.104	0.102
500	0.068	0.069	0.069	0.069	0.078	0.076	0.070	0.073	0.073	0.045	0.085	0.083
1000	0.068	0.067	0.065	0.065	0.073	0.070	0.066	0.067	0.065	0.042	0.076	0.071

SA power detrended



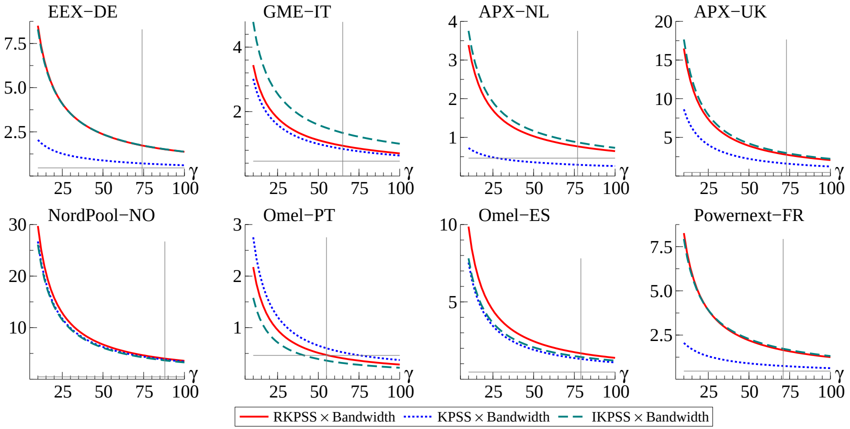
Empirical Example

The three tests are applied to eight time series of European wholesale electricity prices, named after their respective market makers:

- EEX-DE (Germany),
- GME-IT (Italy),
- APX-NL (Netherlands),
- APX-UK (United Kingdom),
- NordPool-NO (Norway),
- Omel-PT (Portugal),
- Omel-ES (Spain),
- Powernext-FR (France).

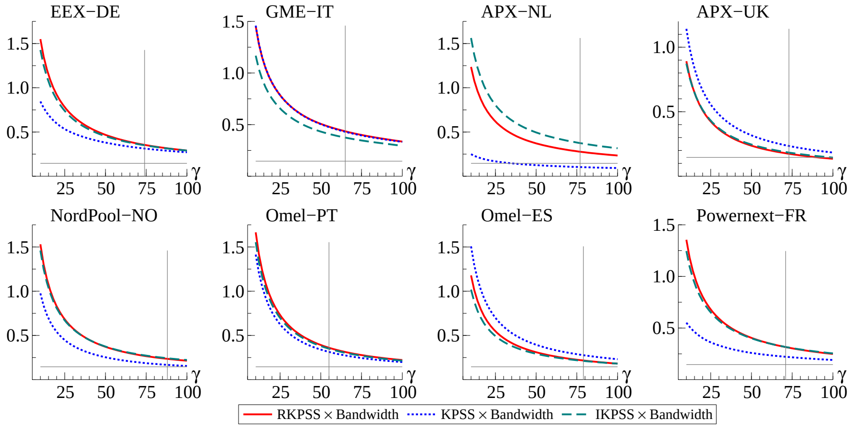
Each observation represents the daily (working day) price at noon. The starting date is different for each series, ranging from the 4th May 1992 of NordPool to the 2nd July 2007 of Omel-PT, while the last observation dates 25th May 2012 for all markets.

Test statistics as functions of the bandwidth parameter γ_T



The horizontal line indicates the 5% critical value and the vertical line denotes the optimal bandwidth for an AR(1) with $\phi = 0.9$.

Detrended test statistics as functions of the bandwidth



The horizontal line indicates the 5% critical value and the vertical line denotes the optimal bandwidth for an AR(1) with $\phi = 0.9$.

Remarks on the empirical application

- KPSS fails to find nonstationarity for APX-NL (both on level and trend),
- the RKPSS statistic is never the closest to zero,
- the RKPSS and IKPSS statistics behave similarly for those cases in which the series are extremely leptokurtic (Powernext-FR, EEX-DE, APX-NL, NordPool-NO, APX-UK).

- New short-memory stationarity test:
  - invariant to monotonic transformations;
  - no moments required;
  - robust to fat-tailed distributions;
  - very good size (typical of rank tests due to distribution freeness);
  - better power than IKPSS and KPSS in many empirically relevant situations (leptokurtosis and positive dependence);
  - maximum loss of ARE w/r to KPSS: 7%.
  - maximum gain of ARE w/r to KPSS: unbounded.
- Extensions:
  - asymptotics under long memory;
  - cointegration rank test of Nyblom & Harvey (2000) type;
  - general score functions of the type used for linear rank statistics: for example Van der Waerden scores

$$a_{T,t} = \Phi^{-1}\left(\frac{R_{T,t}}{T+1}\right),$$

with Φ^{-1} standard normal quantile function and optimality analysis.