Workshop on Statistical modeling for stochastic processes and related fields (online)

This workshop aims to exchange information on the state-of-the-art in statistical modeling for stochastic processes and related fields from theoretical, methodological, and implementation points of view.

Dates: September 27 (Mon) - 30 (Thu), 2021

Time: From 17:00 to 19:30 (in Japan Standard Time (JST))

Online workshop: The style of the workshop is online.

Organizer: Kengo Kamatani (ISM, Tokyo) Hiroki Masuda (Kyushu Uniuversity) Nakahiro Yoshida (University of Tokyo) Masayuki Uchida (Osaka University)

Sponsors: It is supported by the Japan Science and Technology Agency CREST JPMJCR14D7. Program (in Japan Standard Time)

September 27 (Mon) Chair: Nakahiro Yoshida (University of Tokyo)

17:00 - 17:05 Welcome Address by Nakahiro Yoshida (University of Tokyo)

17:05 - 17:40 Nakahiro Yoshida (University of Tokyo) "TBA"

17:40 - 18:15 Yuliia Mishura (Taras Shevchenko National University of Kyiv) *High-Frequency Trading with Fractional Brownian Motion* 

18:20 - 18:55 Ciprian A Tudor (Université Lille 1) "TBA"

18:55 - 19:30 Igor Cialenco (Illinois Institute of Technology) "TBA"

September 28 (Tue) Chair: Hiroki Masuda (Kyushu University)

17:00 - 17:35 Maud Delattre (Universite Paris-Saclay, INRAE, MaIAGE) Statistical inference for discretely observed stochastic differential equations with mixed effects

17:35 - 18:10 Lorenzo Mercuri (University of Milan) "TBA"

18:20 - 18:55 Alexei Kulik (Politechnika Wrocławska) "TBA"

18:55 - 19:30 Hiroki Masuda (Kyushu University) "TBA" September 29 (Wed) [Satellite] Chair: Kengo Kamatani (ISM, Tokyo)

14:55 - 15:30 Goda Takashi (University of Tokyo) Multilevel Monte Carlo methods for efficient nested simulations

15:30 - 16:05 Xin Tong (National University of Singapore) Can Algorithms Collaborate? The Replica Exchange Method and Its Spectral Gap

16:05 - 16:40 Alexandre H. Thiery (National University of Singapore) Exploiting geometry for walking larger steps in Bayesian Inverse Problems

September 29 (Wed) Chair: Kengo Kamatani (ISM, Tokyo)

17:00 - 17:35 Kengo Kamatani (ISM, Tokyo) Scaling limit analysis of some piecewise deterministic Markov processes

17:35 - 18:10 Björn Sprungk (TU Bergakademie Freiberg) "TBA"

18:20 - 18:55 Tony Lelievre (Ecole des Ponts ParisTech) Adaptive importance sampling methods

18:55 - 19:30 Alexandros Beskos (University College London) Manifold Markov chain Monte Carlo methods for Bayesian inference in diffusion models

September 30 (Thu) Chair: Masayuki Uchida (Osaka University)

17:00 - 17:35 Masayuki Uchida (Osaka University) Adaptive test for ergodic diffusion processes from discrete observations

17:35 - 18:10 Mathias Vetter (Christian-Albrechts-Universität zu Kiel) "TBA"

18:15 - 18:50 Markus Bibinger (Julius-Maximilians-Universität Würzburg) Inference on jumps in high-frequency order-price models with one-sided noise

18:50 - 19:25 Yasutaka Shimizu (Waseda University)
M-estimation based on quasi-processes from discrete samples of Lévy processes

19:25 - 19:30 Closing Address by Masayuki Uchida (Osaka University)

## High-Frequency Trading with Fractional Brownian Motion Paolo Guasoni<sup>1</sup>; Yuliya Mishura<sup>2</sup>; Miklós Rásonyi<sup>3</sup>

<sup>1</sup> Dublin City University, School of Mathematical Sciences, Glasnevin, Dublin 9, Ireland, and Boston University, Department of Mathematics and Statistics, 111 Cummington Street, Boston, MA 02215, USA

<sup>2</sup> Taras Shevchenko National University of Kyiv, e-mail: myus@niv.kiev.ua

<sup>3</sup> Alfréd Rényi Institute of Mathematics, Budapest, Hungarian Academy of Sciences

**Abstract:** In the high-frequency limit, conditional expected increments of fractional Brownian motion converge to a white noise, shedding their dependence on the path history and the forecasting horizon, and making dynamic optimization problems tractable. We find an explicit formula for locally mean-variance optimal strategies and their performance for an asset price that follows fractional Brownian motion. Without trading costs, risk-adjusted profits are linear in the trading horizon and rise asymmetrically as the Hurst exponent departs from Brownian motion, remaining finite as the exponent reaches zero while diverging as it approaches one. Trading costs penalize numerous portfolio updates from short-lived signals, leading to a finite trading frequency, which can be chosen so that the effect of trading costs is arbitrarily small, depending on the required speed of convergence to the high-frequency limit.

## Statistical inference for discretely observed stochastic differential equations with mixed effects

Maud Delattre<sup>1</sup>; Valentine Genon-Catalot<sup>2</sup>; Catherine Larédo<sup>3</sup>

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<sup>2</sup> Université de Paris, MAP5, Paris, France

<sup>3</sup> Université Paris-Saclay, INRAE, Jouy-en-Josas, France

## Abstract:

Mixed effects models are popular tools for analyzing longitudinal data from several individuals simultaneously. Individuals are described by the same structural model and the inclusion of some random effects enables the description of the inter-individual variability which is inherent to the data. In stochastic differential equations with mixed effects, the structural model is a set of stochastic differential equations, which are usually observed at discrete times. The estimation of the population parameters, *i.e.* parameters of the random effects distribution, is a delicate problem because there is no closed form for the likelihood function. In order to characterize the statistical inference in such models, we consider a parametric framework with distributions leading to explicit approximate likelihood functions. We investigate the properties of the resulting estimators under the asymptotic framework where both the number of sample paths and the number of observations per path tend to infinity and where observations are collected within a fixed time interval. The estimation methods are assessed on various simulated data sets and then applied to real neuronal data.

## Scaling limit analysis of some piecewise deterministic Markov processes

Kengo Kamatani<sup>1</sup>

<sup>1</sup> Institute of Statistical Mathematics, E-mail: kamatani@ism.ac.jp

## Abstract:

Recently, piecewise deterministic Markov processes have gained interest in the Monte Carlo community in the context of scalable Monte Carlo integration methods. In this talk, we will discuss scaling limits for some piecewise deterministic Markov processes. We will describe these results using multiscale analysis, which is a useful technique for this purpose.

This is joint work with J. Bierkens (TU Delft) and G. O. Roberts (Warwick).

## Adaptive importance sampling methods

Tony Lelièvre<sup>1</sup>

<sup>1</sup> Ecole des Ponts ParisTech, E-mail: tony.lelievre@enpc.fr

## Abstract:

We will present the general strategy of free energy adaptive importance sampling methods. These are very efficient algorithms to sample multimodal distributions with an information on the modes given by a so-called collective variable which indexes the modes. We will also discuss recent techniques to build this collective variable.

## References:

- T. Lelièvre, M. Rousset and G. Stoltz, Long-time convergence of an Adaptive Biasing Force method, Nonlinearity, 21, 1155-1181 (2008).

- G. Fort, B. Jourdain, T. Lelièvre and G. Stoltz, Convergence and efficiency of adaptive importance sampling techniques with partial biasing, Journal of Statistical Physics, 171(2), 220-268 (2018).

- B. Jourdain, T. Lelièvre and P.A. Zitt, Convergence of metadynamics: discussion of the adiabatic hypothesis, to appear in Annals of Applied Probability.

- Z. Belkacemi, P. Gkeka, T. Lelièvre and G. Stoltz, Chasing Collective Variables using Autoencoders and biased trajectories, <u>https://arxiv.org/abs/2104.11061</u>.

# Manifold Markov chain Monte Carlo methods for Bayesian inference in diffusion models

Matthew M Graham<sup>1</sup>; Alexandre H Thiery<sup>2</sup>; Alexandros Beskos<sup>3</sup>

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## Abstract:

Bayesian inference for nonlinear diffusions, observed at discrete times, is a challenging task that has prompted the development of a number of algorithms, mainly within the computational statistics community. We propose a new direction, and accompanying methodology – borrowing ideas from statistical physics and computational chemistry – for inferring the posterior distribution of latent diffusion paths and model parameters, given observations of the process. Joint configurations of the underlying process noise and of parameters, mapping onto diffusion paths consistent with observations, form an implicitly defined manifold. Then, by making use of a constrained Hamiltonian Monte Carlo algorithm on the embedded manifold, we are able to perform computationally efficient inference for a class of discretely observed diffusion models. Critically, in contrast with other approaches proposed in the literature, our methodology is highly automated, requiring minimal user intervention and applying alike in a range of settings, including: elliptic or hypo-elliptic systems; observations with or without noise; linear or non-linear observation operators. Exploiting Markovianity, we propose a variant of the method with complexity that scales linearly in the resolution of path discretisation and the number of observation times. Example Python code is given at git.io/m-mcmc.

### Adaptive test for ergodic diffusion processes from discrete observations

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### Abstract :

We consider statistical hypothesis testing problems for multidimensional ergodic diffusion processes based on high frequency data. Kitagawa and Uchida (2014, JSPI) investigated the joint testing problem for the null hypothesis with respect to (w.r.t.) both the diffusion and drift parameters. In this talk, we propose the two-step testing method for the diffusion and drift parameters as follows: (i) test the null hypothesis H 1 w.r.t. only the diffusion parameter versus the alternative hypothesis A 1 (not H 1). (ii) test the null hypothesis H 2 w.r.t only the drift parameter versus the alternative hypothesis A 2 (not H 2). We call this test the adaptive test. To construct the test statistics of the adaptive tests, we utilize the adaptive maximum likelihood type estimators of both the diffusion and drift parameters and provide three kinds of the adaptive test statistics: likelihood ratio type, Wald type and Rao's score type adaptive test statistics. It is proved that the adaptive test statistics converge in distribution to the chi-squared distribution under the null hypothesis and the adaptive tests are consistent against the alternatives. Moreover, it is shown that the adaptive test statistics converge in distribution to the non-central chisquared distribution under local alternatives. Some examples are also given and simulation studies show the asymptotic behavior of the three kinds of the adaptive test statistics.

## Inference on jumps in high-frequency order-price models with one-sided noise Markus Bibinger<sup>1</sup>

<sup>1</sup> Julius-Maximilians-Universität Würzburg, E-mail: markus.bibinger@uni-wuerzburg.de

**Abstract:** For high-frequency intra-day price processes it is well-known that market microstructure dilutes the underlying dynamics of a semi-martingale model. Different to the classical model for traded prices with additive, centred market microstructure noise, we consider a stochastic boundary model with one-sided noise for best-ask prices from the limit order book. We construct methods to estimate, locate and test for jumps in this model. The different structure of our irregular noise leads us to statistics based on local minima instead of local averages, which have been used for regular noise in the literature. We first discuss inference for a possible jump at some given (stopping) time. We provide a local test and show that we can consistently estimate price jumps. We develop a global test for jumps based on extreme value theory. We establish the asymptotic distribution of a test statistic under the null hypothesis and consistency under the alternative hypothesis. The rate of convergence for local alternatives is faster than in the regular noise model what allows the identification of smaller jumps based on discrete observations. In the process, we establish central limit theorems for spot volatility estimation.

#### M-estimation based on quasi-processes from discrete samples of Lévy processes

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### Abstract:

Let  $X = (X_t)_{t\geq 0}$  be a Lévy process starting from  $X_0 = u$ , which is a random element values in  $\mathbb{D}_{\infty}$  that is a space of càdlàg functions on  $\mathbb{R}_+ := [0, \infty)$  with the Skorokhod topology. Given a function  $h_{\vartheta}(x) = h(x, \vartheta) : \mathbb{D}_{\infty} \times \Theta \to \mathbb{R}$ , our interest is an *M*-estimation for a parameter defined as

$$\vartheta_0 = \arg\min_{\vartheta \in \Theta} \mathbb{E}[h_\vartheta(X)]$$

from discrete observations of  $X = (X_t)_{t \ge 0}$ .

An example of  $h_{\vartheta}$  from insurance and finance is the following functional:

$$h_{\vartheta}(x) = \int_{0}^{\tau_{\vartheta}(x)} e^{-rt} U_{\vartheta}(x_{t}) \,\mathrm{d}t, \quad x = (x_{t})_{t \ge 0} \in \mathbb{D}_{\infty},$$

where  $\tau_{\vartheta}(x) := \inf\{t > 0 \mid x_t < \vartheta\}$  and  $U_{\vartheta} : \mathbb{R} \to \mathbb{R}$  for each  $\vartheta \in \Theta$ . Such a problem typically appears in the 'ruin' or 'default' analysis. The difficulty of this problem is that  $h_{\vartheta}$  depends on whole the path of X, and that the random time  $\tau_{\vartheta}(X)$  is usually unobservable in practice. Therefore it is hard to construct an estimator of  $\mathbb{E}[h_{\vartheta}(X)]$  from discrete samples of X.

To overcome the difficulty, we produce "quasi-paths" by shuffling increments of the Lévy process, we call it *quasi-processes*. By shuffling the order of increments randomly and connecting them as step functions, we can produce many quasi-paths of the Lévy process, and we can show that the distribution of the quasi-process converges weakly to the true process in  $\mathbb{D}_{\infty}$ -space under a high-frequent sampling scheme. In the talk, we will confirm the convergence result in some numerical experiments.

Using such a resampling technique like the bootstrap, we can estimate the objective functional  $\mathbb{E}[h_{\vartheta}(X)]$  as if we used the Monte Carlo method, and it is available as a contrast function to estimate  $\vartheta_0$ . We further show that the *M*-estimator based on these quasi-processes can be consistent and asymptotically normal under suitable conditions. Although we shall give only theoretical results in this talk, an example from dividend strategy in insurance is discussed in an another paper Shimizu and Shiraishi (2021).

## References

[1] Shimizu, Y. and Shiraishi, H. (2021). Semiparametric Estimation of Optimal Dividend Barrier for Spectrally Negative Lévy Process, *preprint*.